Parametric co-amplification of waves on a rivulet

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When studying dynamical systems, one usually considers either *external* (additive) or *parametric* forcing, each having its distinct characteristics. We investigate an original and intriguing hydrodynamic instability, where imposing an external forcing results in a parametric destabilization.

We study the behaviour of a fluid filament (a *rivulet*) flowing in an air-filled Hele-Shaw cell. In this seemingly simple system, reciprocal interactions between the flow inside the rivulet and the geometry of its free surface can lead to complex behaviour [1]. Two kind of perturbations are observable: transverse waves correspond to deformations of the rivulet path, and longitudinal waves correspond to modulations of its width (fig. 1). In the parameter range we adopt, both of these perturbations are linearly damped and the rivulet usually follows a straight path, flowing down with constant width.

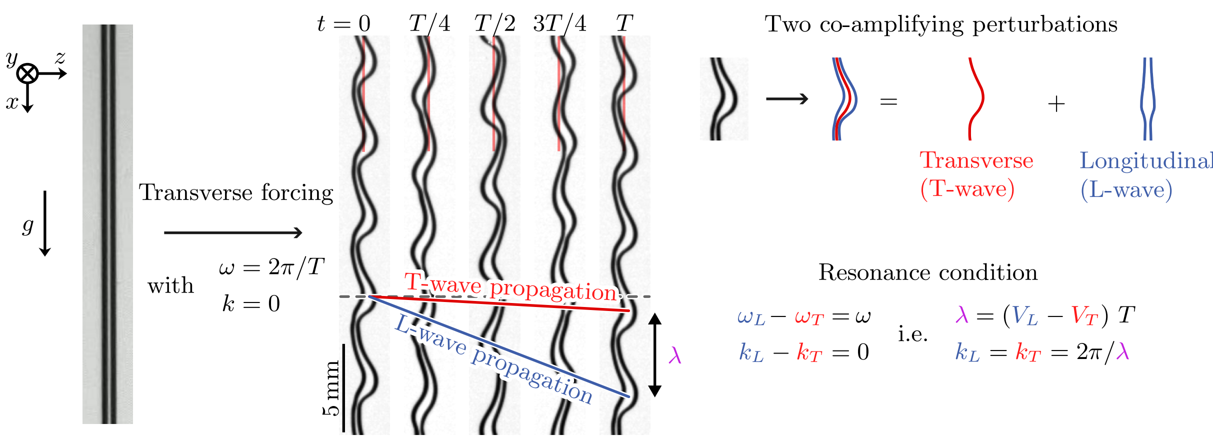

Figure 1. The pattern exhibited by the rivulet when subject to a spatially homogeneous transverse excitation. We observe strong transverse and longitudinal deformations, which propagate at different phase velocities $V_{T,L}$.

We impose an external, homogeneous, harmonic forcing to the rivulet. When the forcing amplitude exceeds a given threshold, the rivulet responds nonlinearly, adopting a peculiar pattern shown on fig. 1. We show that this behaviour results from a parametric co-amplification due to resonant interactions between transverse and longitudinal waves on the rivulet [2]. This is reminiscent of the interaction between counterpropagative waves in the Faraday instability [3]. The amplified waves satisfy a resonance condition, which selects a precise wavelength. Using depth-averaged Navier-Stokes equations, we derive a model for the synchronous co-amplification. This allows us to explain the physical origin of the phenomenon and quantitatively describe its spatio-temporal features. We present extensive experimental evidence confirming the relevance of our model.

References

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