Scale relativity applied to geophysical turbulence

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Abstract

A new formalism has been developed for the study of turbulence using the scale relativity framework [1]. In this work, we extend the previous studies. Precisely, we discuss the application of the scale-relativity approach applied to a turbulent fluid in rotation. It explores the transformation of the time derivative of the Navier-Stokes equation in the usual x-space into a Schrödinger-like equation in velocity space. This transformation involves introducing an external vectorial field to account for rotation and a local Velocity Harmonic Oscillator (VHO) potential in velocity space. The coefficients of the VHO potential are determined by second-order x-derivatives of the pressure. Then we derive formulae for the Probability Distribution Functions (PDF) of velocity and acceleration. Thus, predictions are then compared with data from 'oceanic drifters' velocity measurements. We show a good agreement between the predicted acceleration PDF and the observed data from oceanic drifters [2].

Basic Concepts

The theory of scale relativity and fractal space-time is based on relaxing the assumption of space smoothness and differentiability, it means a fractal space. This theory has been recently applied to the problem of turbulence in fluid mechanics. The key idea for this application consists of working in velocity-space instead of position-space, i.e., of using the velocity as basic coordinate. This suggestion was motivated by the Kolmogorov (K41). However, one important field of application of turbulence theory is geophysics for which the presence of a Coriolis need to be taken into account.

Models

For geophysical turbulence, we define a "geopotential" $\widetilde{\Phi}_v(v)$ in v-space (as we consider the existence of cascade of eddies described as a sum of oscillators. then, Schrödinger-Coriolis (SC) equation written in v-space:

$$\left(\mathcal{D}_v \nabla_v - \frac{1}{2} i K_v(v)\right)^2 \psi_v + i\mathcal{D}_v \frac{\partial \psi}{\partial t} - \frac{1}{2}\widetilde{\Phi}(v)\psi_v = 0$$

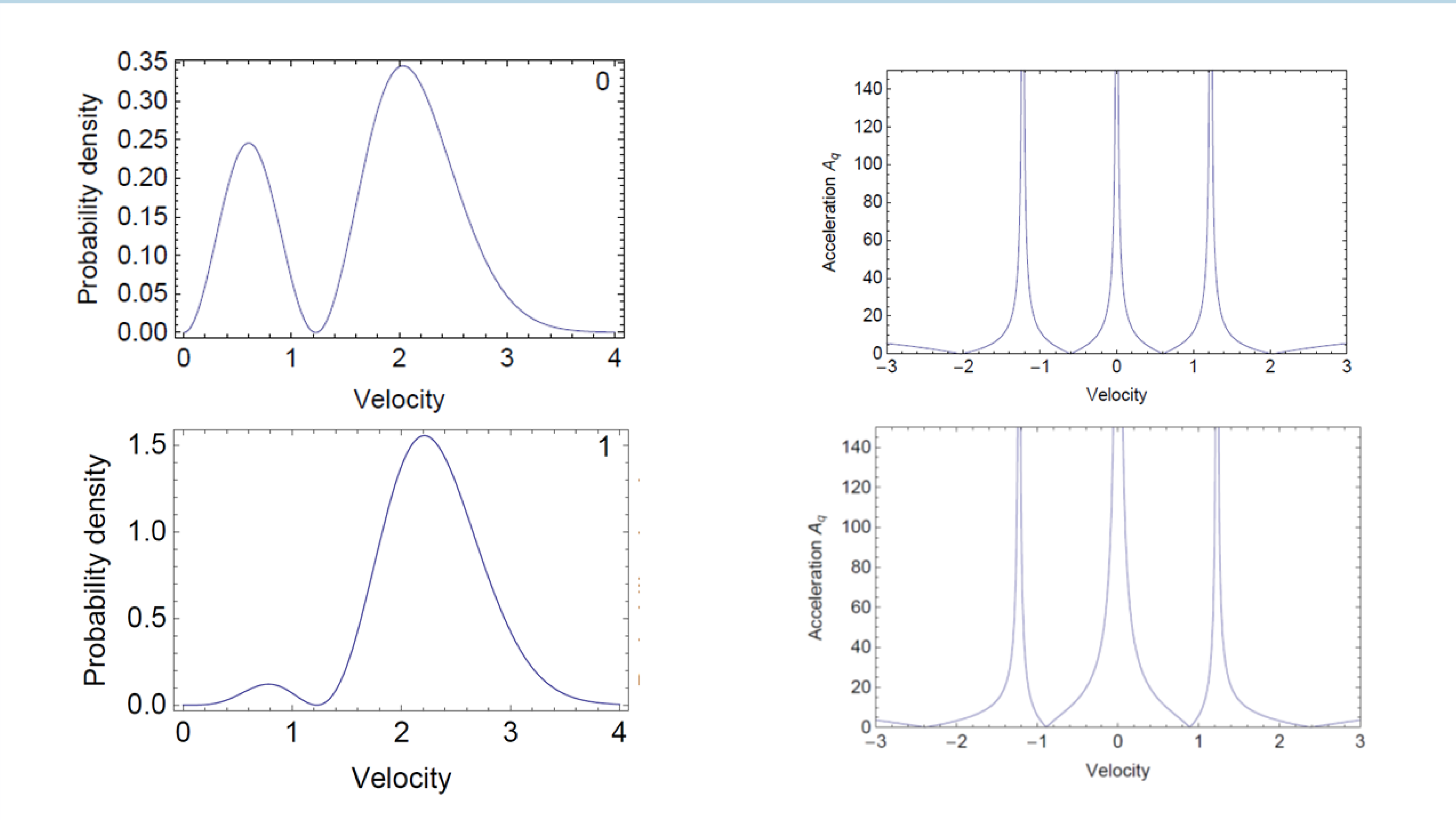
Since the emergence of a SC equation relies on the scaling of the inertial range itself which serves as microscopic theory to construct it, it strictly applies only to the largest integral scales of this range. At this maximal scale, the geostrophic potential is reduced to an oscillator which we assume to be harmonic with frequency ω. Then,the Hamiltonian reads: H$=(p_{v_x} + \Omega\, v_y)^2 + (p_{v_y} - \Omega\, v_x)^2 + \frac{1}{2}\omega^2((v_x - v_{0x})^2 + (v_y - v_{0y})^2 \sum) + p_{v_z}^2 + \frac{1}{2}\omega_z^2 v_z^2, where p_{v_k} = i_{v_k}\nabla_{v_k}$, accounting for a possible anisotropy in the diffusion coefficient.

Solutions to Schrödinger-Coriolis-Coriolis equation in v-space

We derive various solutions of the Schrödinger equation including a Coriolis force expressed in terms of the vectorial field K_v.

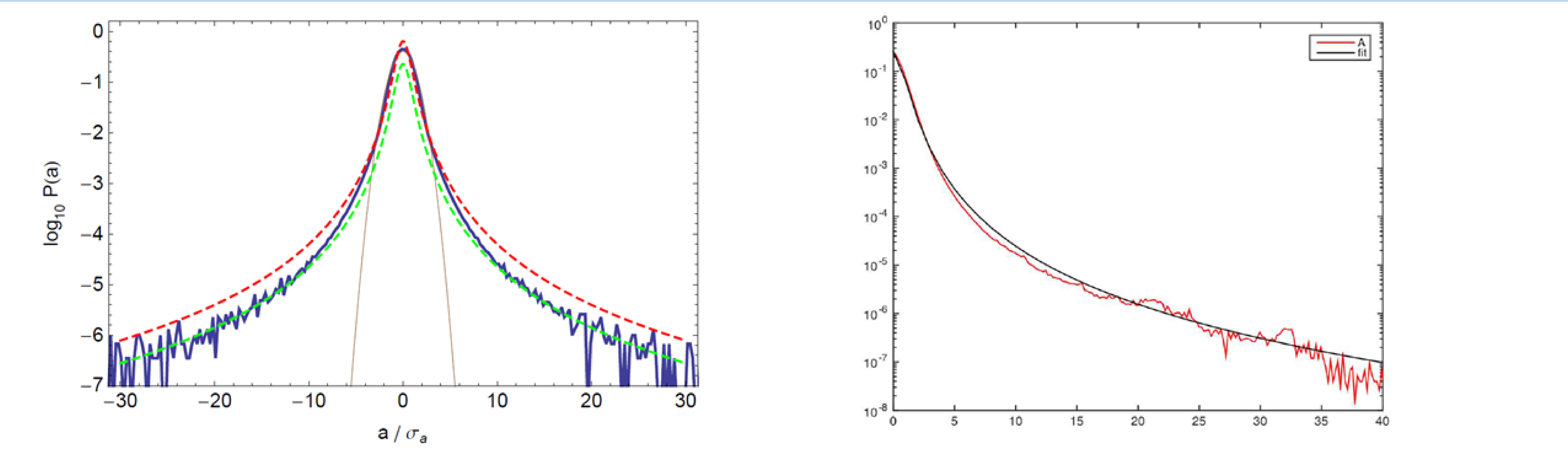
We have shown that, in Lagrangian turbulence experiments, one can locally describe the oscillatory motion of a particle swept away in an eddy as an harmonic oscillator (HO, possibly damped: DHO). This can be accounted for by introducing an HO potential in the motion equation.

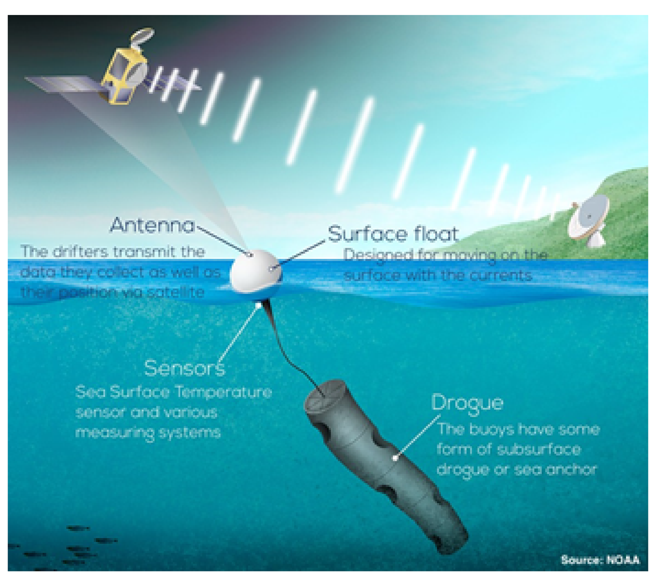
We assume that the same approach can be used in the geostrophic case. An exact solution can be found using the eigenvalue for the z component of the angular momentum operator L_z, which intervenes here in the combination $2\,\Omega\,(v_x p_{v_y} - v_y p_{v_x})$. Therefore we take polar coordinates in 2D v-space $(v,\ \varphi)$ and v_z in the z direction, and we obtain for the stationary equation $H\psi = E\psi$.

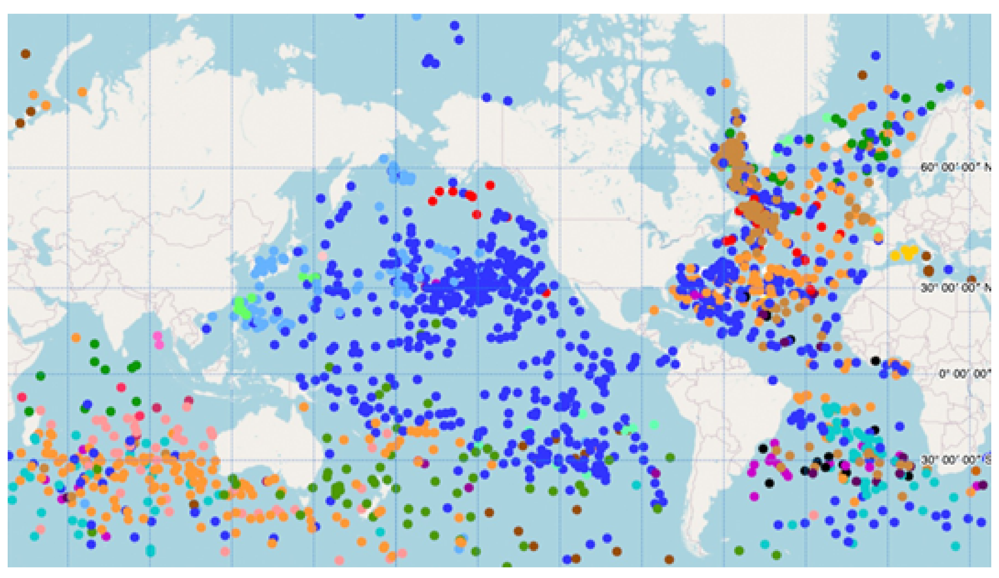


Test with geophysical data

We simulate the probability density function (PDF) of velocity for a harmonic oscillator (left figure below), particularly focusing on the n = 3 level for examining the effects of the Coriolis force on this system. Acceleration PDF obtained from a numerical simulation of a quantum harmonic oscillator (10^7 points). The blue thick curve is the PDF obtained in the presence of a Coriolis force. The red dashed curve is the PDF obtained in the absence of a Coriolis force with the same normalization, which is very well fitted in the acceleration range shown ($\pm 30\sigma_a$). The green dashed curve is the same no-Coriolis PDF, but now fitted on the large tails of the Coriolis PDF. The brown thin line is a Gaussian curve with the same standard deviation σ_a.







Then we apply SR to loopers (i.e., drifters with looping trajectories that complete at least two orbits). They have been identified in Lagrangian trajectory data by an automatic algorithm. This algorithm has been applied to the Global Drifter Program data set, and over 15,000 looping trajectory segments have been identified worldwide. The global drifter acceleration PDF is well fitted by an $1/a^4$ up to $\approx 40\sigma$, as can be seen in the right curve. We prove excellent agreement with the theoretical expectation from the scale-relativity description of turbulent flows. Studies about the PDF of accelerations of loopers in function of latitude have been made. (owing to the fact that the Coriolis force varies with latitude, vanishing at the equator and being maximal at the poles). All the PDFs show a common exponent $P_a(a) \sim a^4$ up to large values of the acceleration, in agreement with the laboratory data and with the theoretically expected value for $l_z = 0$.

References

[1] Laurent Nottale, Thierry Lehner Turbulence and scale relativity. Physics of Fluids 1 October 2019; 31 (10): 105109. https://doi.org/10.1063/1.5108631

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