Weakly nonlinear theory of Echo State Networks

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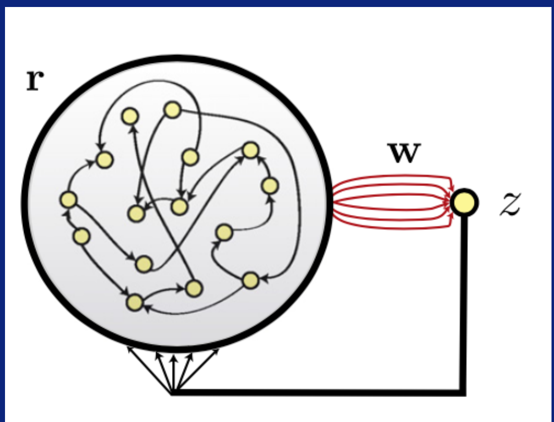
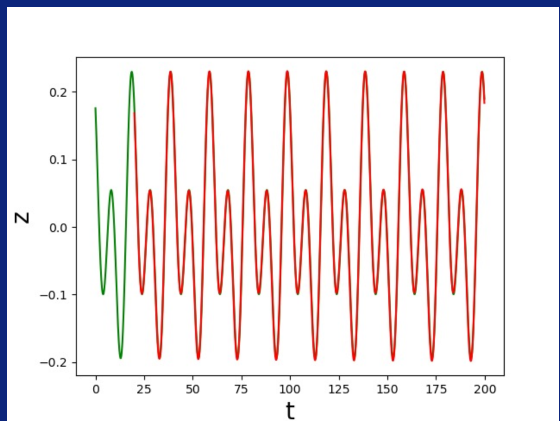
$$\frac{d\mathbf{x_i}}{dt} = -\mathbf{x_i} + g\sum_j M_{ij} r(x_j) + b_i \sum_j w_j r(x_j)$$

« Reservoir » of randomly connected units (neurons,electronic circuits, mechanical networks, ….)





« Magic » : one can learn the feedback **w** so that the **autonomous dynamics** produce any desired function : $z(t) = \sum_j w_j r[x_j(t)])$

Jaeger and Haas, 2004; Sussillo & Abbott, 2009;...

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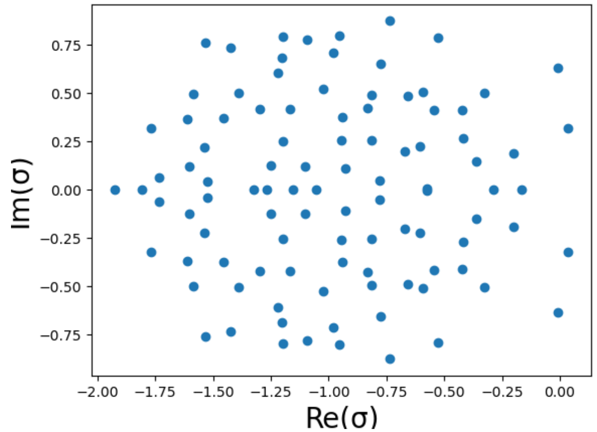
- **How does this work?** not well-understood in general.

- **Our contribution** : a detailed explanation in the regime when the starting **network is stable** and the **feedback z(t)** is of **small amplitude**.



- **Three steps :**

i) **Linear level learning** : use the rank 1 feedback to place a finite number of modes close of the imaginary axis => approximate Fourier decomposition of the function.

$$z(t) = [Z_1 \exp(i\omega t) + Z_2 \exp(i2\omega t) + c.c]$$

ii) **Weakly linear level** : the interaction of these slow modes can be described by amplitude equations.

$$\frac{dZ_1}{dt} = \lambda_1 Z_1 + (g_{11}|Z_1|^2 + g_{21}|Z_2|^2)Z_1$$
$$\frac{dZ_2}{dt} = \lambda_2 Z_2 + (g_{12}|Z_1|^2 + g_{22}|Z_2|^2)Z_2$$

iii) **Weakly linear level learning**: amounts to refining the feedback **w** and give eigenvalues (small) real parts which, with the nonlinear terms, select appropriate modes/Fourier amplitudes (and phases).