Turbulence modeling in the QR space

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Coarse-grained Navier-Stokes problem (not closed)

$$\bar{\boldsymbol{u}}^q(\boldsymbol{x},t) = \int G^q(\boldsymbol{r})\boldsymbol{u}(\boldsymbol{x}-\boldsymbol{r},t)d\boldsymbol{r}, \quad \widetilde{G^q}(\boldsymbol{k}) = exp\left[-\frac{k^2}{2q^2}\right]$$

$$\partial_t \bar{\boldsymbol{u}}^q + (\bar{\boldsymbol{u}}^q \cdot \boldsymbol{\nabla})\bar{\boldsymbol{u}}^q = -\boldsymbol{\nabla}\bar{p}^q - \boldsymbol{\nabla}\cdot\tau^{\boldsymbol{q}} + \nu\boldsymbol{\nabla}^2\bar{\boldsymbol{u}}^q + \bar{\boldsymbol{f}}^q$$

$$\boldsymbol{\nabla}\cdot\bar{\boldsymbol{u}}^q = 0$$

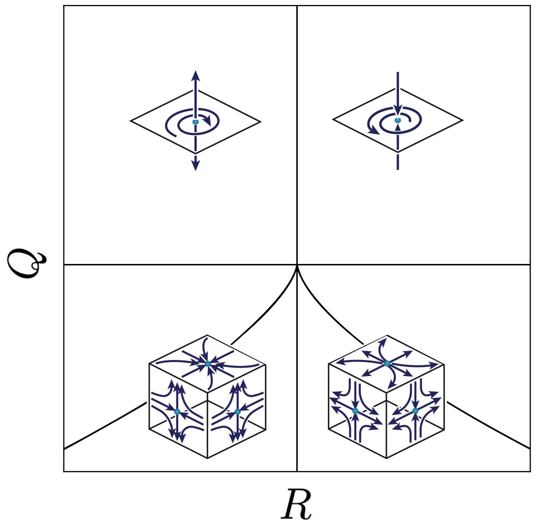
SubGrid-Scale stress tensor (to be modeled)

$$\tau^{\boldsymbol{q}}{}_{ij} = \overline{u_i u_j}^q - \bar{u_i}^q\,\bar{u_j}^q$$

Invariants of the Velocity Gradient Tensor

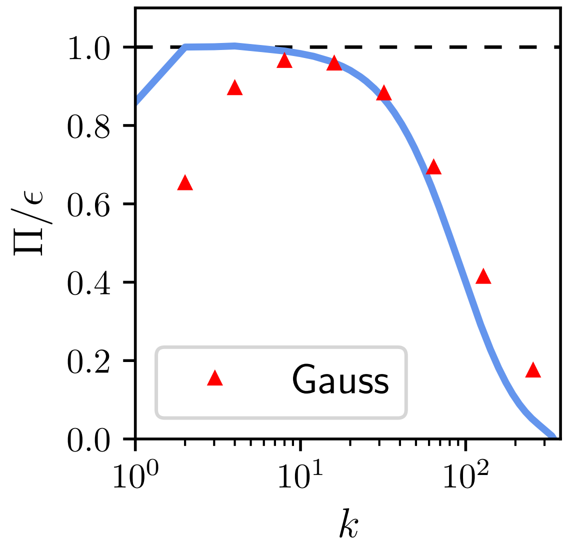
$$Q^q(\boldsymbol{x},t) = -\frac{1}{2}\frac{\partial\bar{u_i}^q}{\partial x_j}\frac{\partial\bar{u_j}^q}{\partial x_i}$$

$$R^q(\boldsymbol{x},t) = -\frac{1}{3}\frac{\partial\bar{u_i}^q}{\partial x_j}\frac{\partial\bar{u_j}^q}{\partial x_l}\frac{\partial\bar{u_l}^q}{\partial x_i}$$



Direct Numerical Simulations
Setup: box periodic boundary conditions

$$L = 2\pi, \quad N = 1024, \quad \nu = 5\cdot 10^{-4}, \quad \epsilon = 1.$$



Local energy flux

$$\Pi^q(\boldsymbol{x},t) = -\tau^{\boldsymbol{q}}{}_{ij}\frac{\partial\bar{u_i}^q}{\partial x_j}$$

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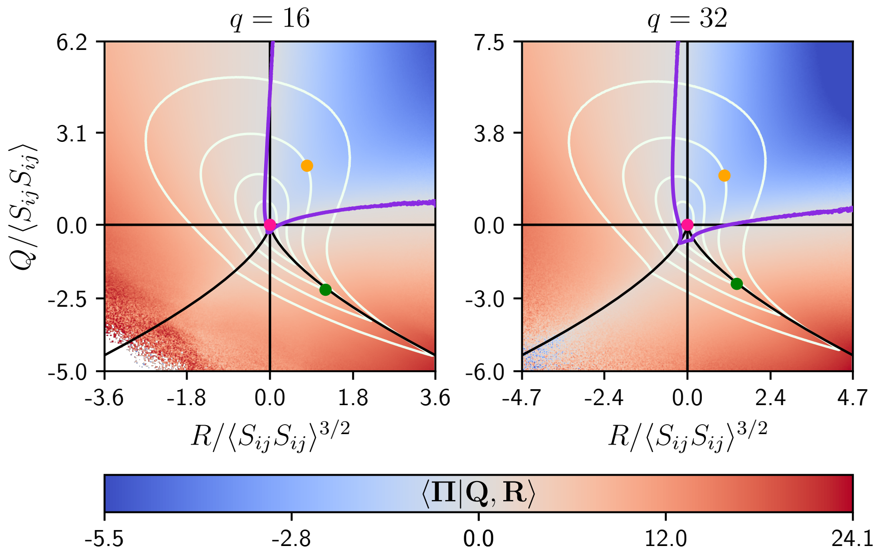

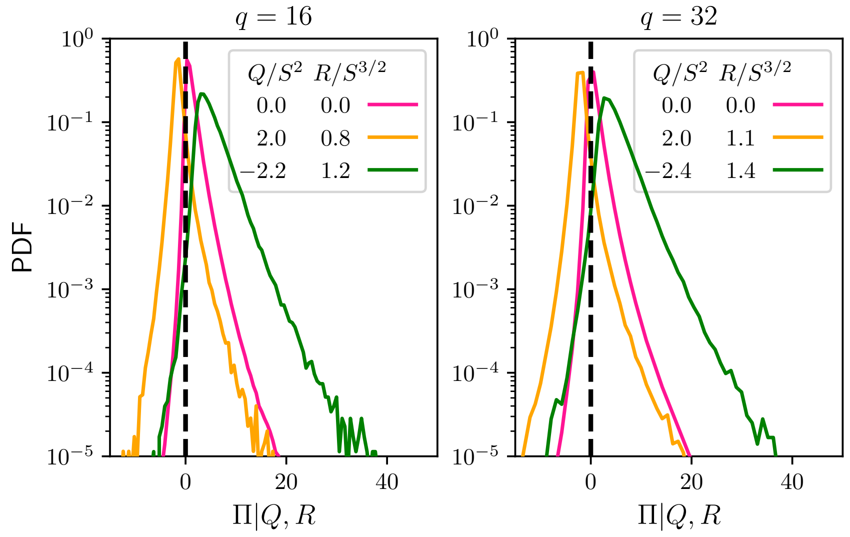

Clark model

$$\tau^{q}{}_{ij} \sim \frac{1}{q^2}\frac{\partial \overline{u_i}^{q}}{\partial x_l}\frac{\partial \overline{u_j}^{q}}{\partial x_l}$$

(local-scale approximation)