SEIRS – SEID epidemic spreading model in complex networks with mortality and resetting

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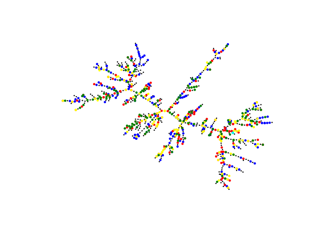
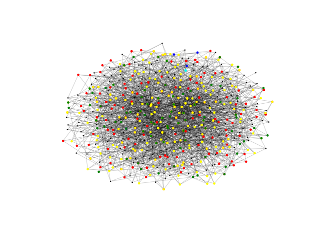
Stochastic compartment model





S : susceptible (for infection)
E : exposed, infected but not infectious ; random sojourn time t_E
I : infected and infectious ; random sojourn time t_I
R : recovered and immune; random sojourn time t_R
D : dead (invisible)

sojourn times t_E, t_I, t_R, t_M drawn from specific PDFs (here Gamma dist.)
sojourn time in D $= \infty$
survival time span t_M counted from the moment of breakout of infection : mortality during sojourn in I

Infection rule :

If S meets I then S gets infected (with prob. P_{inf})

Mortality: only I walkers may die
Survival pathway (for $t_M > t_I$) : SEIRS

$$S \rightarrow E \rightarrow I \rightarrow R \rightarrow S$$

Death pathway of infected walker (for $t_M < t_I$) : SEID

$$S \rightarrow E \rightarrow I \rightarrow D$$

Evolution equations with mortality

$$
\begin{aligned}
\frac{d}{dt}S(t) &= -\mathcal{A}(t) \\
&+\langle (J_0\delta(t-t_I-t_R)+\mathcal{A}(t-t_E-t_I-t_R))\Theta(t_M-t_I)\rangle + R_0\langle\delta(t-t_R)\rangle \\
\frac{d}{dt}E(t) &= \mathcal{A}(t)-\langle\mathcal{A}(t-t_E)\rangle \\
\frac{d}{dt}J(t) &= \langle\mathcal{A}(t-t_E)\rangle - \langle\{J_0\delta(t-t_I)+\mathcal{A}(t-t_E-t_I))\Theta(t_M-t_I)\}\rangle \\
&-\langle\{J_0\delta(t-t_I)+\mathcal{A}(t-t_E-t_I))\Theta(t_I-t_M)\}\rangle \\
\frac{d}{dt}R(t) &= -R_0\langle\delta(t-t_R)\rangle + \langle (J_0\delta(t-t_I)+\mathcal{A}(t-t_E-t_I))\Theta(t_M-t_I)\rangle \\
&-\langle (J_0\delta(t-t_I-t_R)+\mathcal{A}(t-t_E-t_I-t_R))\Theta(t_M-t_I)\rangle
\end{aligned}
$$

$\langle\ldots\rangle$ means averaging with respect to independent random waiting times t_E, t_I, t_R, t_M which we assume to be Gamma-distributed.

Infection rate: $\mathcal{A}(t)=\beta S(t)J(t)$ only I walkers may infect S walkers

Initial conditions $[S(t)+J(t)+R(t)]\Big|_{t=0}=1,\ D(0)=E(0)=0$

Spreading with mortality – click to visualize

Spreading without mortality;
emergence of endemic state – click to visualize

$$S_e = \frac{1}{\mathcal{R}_0}, \qquad \mathcal{R}_0 = \beta \langle t_I \rangle,$$

$$E_e = \frac{\mathcal{R}_0 - 1}{\mathcal{R}_0} \frac{\langle t_E \rangle}{\langle T \rangle}$$

$$J_e = \frac{\mathcal{R}_0 - 1}{\mathcal{R}_0} \frac{\langle t_I \rangle}{\langle T \rangle} \qquad \langle T \rangle = \langle t_E + t_I + t_R \rangle$$

$$R_e = \frac{\mathcal{R}_0 - 1}{\mathcal{R}_0} \frac{\langle t_R \rangle}{\langle T \rangle}$$

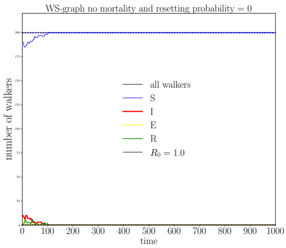
$R_0 > 1$ basic reproduction number (for $R_0 < 1$ no spreading)

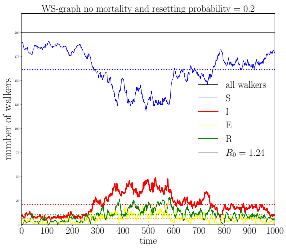
$\langle t_C \rangle$ mean latency time; $\langle t_I \rangle$ mean infection time; $\langle t_R \rangle$ mean immunity time

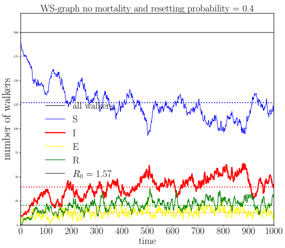
Animated simulation Videos

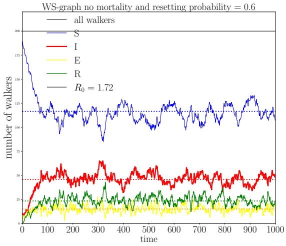
Left: Spreading with mortality
Right: Spreading without mortality – emergence of endemic equilibrium

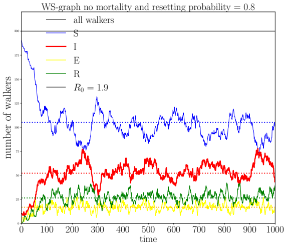
Effect of resetting on spreading without mortality

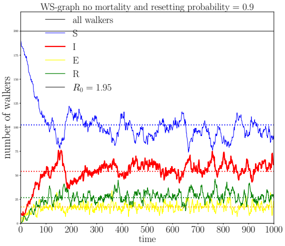












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