Dynamics of wave spectrum evolution due to 4-wave non-linear interactions evaluated with the quasi-exact Gaussian Quadrature Method (GQM)



## Context - TOMAWAC code ( <u>http://www.opentelemac.org/</u>):

Sea state evolution

Numerical resolution

## Modelling:

- Balance equation for wave action *N*
- Mathematical models for energy input, transfer and dissipation terms

$$\frac{\partial N}{\partial t} + \frac{\partial (\dot{x}N)}{\partial x} + \frac{\partial (\dot{y}N)}{\partial y} + \frac{\partial (\dot{k}_x N)}{\partial k_x} + \frac{\partial (\dot{k}_y N)}{\partial k_y} = Q(k_x, k_y, x, y, t)$$

Focus on the energy transfer due to 4-waves interaction (term  $Q_{nl}$ ):  $Q_{nl}^{exact} = \iiint \sigma_4.G.\delta (\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \delta (\sigma_1 + \sigma_2 - \sigma_3 - \sigma_4)$ 

 $\left[\frac{F(\boldsymbol{k}_{1})}{\sigma_{1}}\frac{F(\boldsymbol{k}_{2})}{\sigma_{2}}\left(\frac{F(\boldsymbol{k}_{3})}{\sigma_{3}}+\frac{F(\boldsymbol{k}_{4})}{\sigma_{4}}\right)-\frac{F(\boldsymbol{k}_{3})}{\sigma_{3}}\frac{F(\boldsymbol{k}_{4})}{\sigma_{4}}\left(\frac{F(\boldsymbol{k}_{1})}{\sigma_{1}}+\frac{F(\boldsymbol{k}_{2})}{\sigma_{2}}\right)\right]d\boldsymbol{k}_{1}d\boldsymbol{k}_{2}d\boldsymbol{k}_{3}$ 

- Understanding its role in sea state physics
- Search of the most efficient numerical model to solve it between:
  - Approximate methods (DIA, mDIA)
  - Quasi-exact methods (GQM)

## At a given location:

- Understanding climatology through statistical processing (*hindcast*)
- Short-term predictions

direction

frequency 1/2



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## Study presentation

- Objectives:
  - Understand the role of quadruplet non-linear interactions in the wave spectrum dynamics
  - Comparison of results obtained with different numerical methods
- Beginning from <u>arbitrary initial condition</u>, we let the wave spectrum evolve under the only effect of  $Q_{nl}$  term (Hasselmann's kinetic equation) for a long physical time (roughly one year) in deep water conditions
- $Q_{nl}$  is computed with GQM, mDIA and DIA
- The results are compared to the predictions from the weak turbulence theory



- Good agreement with theoretical results (weak turbulence theory)
- Once reached a near equilibrium state, self-similarity of the solution
- For GQM results, convergence to the same final spectrum regardless the initial condition (fixed initial steepness)



FIG - Evolution of frequency spectrum in the short-term (left) and long-term (right) for beginning stepshaped condition simulated with GQM