

## Data-drive dominant balance search algorithm

The concept of **dominant balance** is that whilst for any differential equation, the sum of terms will always equal zero,

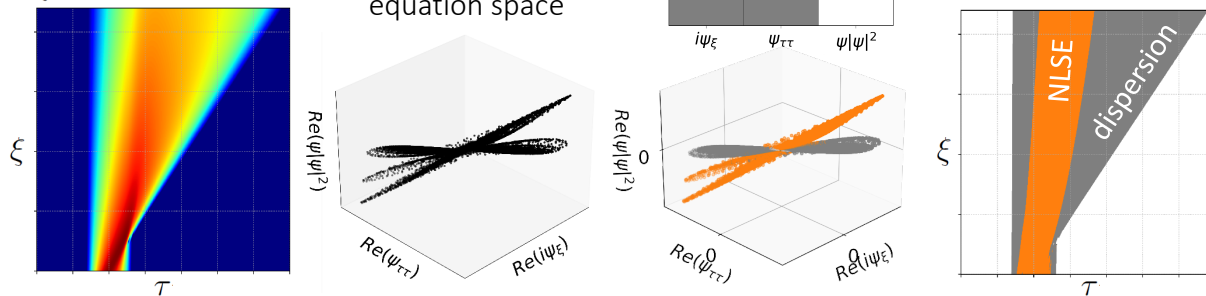
$$\sum_{k=1}^K f_k(\psi, \psi_\xi, \psi_\tau, \dots, \psi^2, \psi\psi_\xi, \psi\psi_\tau, \dots, \psi_{\xi\xi}, \psi_{\tau\tau}, \dots) = 0$$

different combinations of terms may dominate the equality in different spatio-temporal regions.

### Machine learning framework:

- (1) Computing the spatio-temporal evolution in the term-by-term “equation space”
- (2) Partition of the dynamics via Gaussian mixture model clustering
- (3) Sparse principal component analysis / Combinatorial model selection
- (4) Re-mapping the identified dominant balances to the original domain

$$i\psi_\xi + \psi_{\tau\tau} + |\psi|^2\psi = 0$$

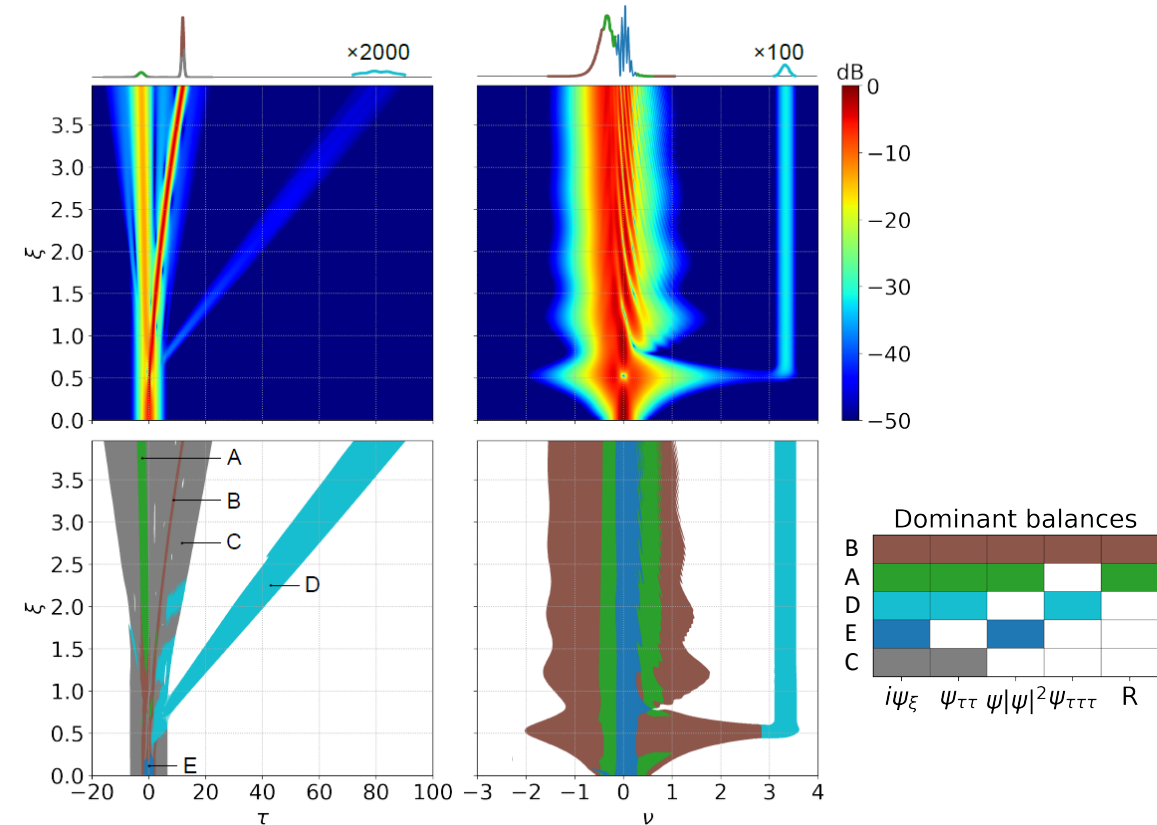


### References

1. J. L. Callaham, J. V. Koch, B. W. Brunton, J. N. Kutz, S. L. Brunton, Nat. Com., 12, 1016 (2021)
2. A. V. Ermolaev, C. Finot, G. Genty, J. M. Dudley, Opt. Lett., 49, 4202-4205 (2024)
3. J. M. Dudley, G. Genty, S. Coen, Rev. Mod. Phys., 78, 1135-1184 (2006)

## An example – soliton fission dynamics

$$i\psi_\xi + \psi_{\tau\tau} + i\delta\psi_{\tau\tau\tau} + |\psi|^2\psi + \rho\psi (h_R * |\psi|^2) = 0$$



### Other examples:

- Optical wave breaking
- Riemann wave propagation
- Soliton fission induced by third order dispersion