Numerical simulations of internal gravity wave turbulence

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Stratified flows support the propagation of internal gravity waves, which are important in many geophysical applications such as mixing in the deep ocean [1]. Yet, simulating waves directly is too expensive for climate modeling due to the space and time scale separation, so their effects are parameterized. In the limit of weak nonlinearity, the Weak Wave Turbulence theory describes the evolution of the system with a kinetic equation

$$\dot{n}_{\boldsymbol{k}} = \int \left[\mathcal{R}^{\boldsymbol{k}}_{12} - \mathcal{R}^{1}_{\boldsymbol{k}2} - \mathcal{R}^{2}_{\boldsymbol{k}1}\right] \, \mathrm{d}^3\boldsymbol{k}_1 \, \mathrm{d}^3\boldsymbol{k}_2 + f_{\boldsymbol{k}} - d_{\boldsymbol{k}},$$
$$\mathcal{R}^{\boldsymbol{k}}_{12} = 4\pi \, \delta(\boldsymbol{k} - \boldsymbol{k}_1 - \boldsymbol{k}_2) \, \delta(\omega_{\boldsymbol{k}} - \omega_1 - \omega_2) \, |V^{\boldsymbol{k}}_{12}|^2 \, \left(n_1 n_2 - n_{\boldsymbol{k}} n_1 - n_{\boldsymbol{k}} n_2\right),$$

$n_{\boldsymbol{k}}$ being the wave action spectrum, $f_{\boldsymbol{k}}$ the forcing, $d_{\boldsymbol{k}}$ the dissipation, $\omega_{\boldsymbol{k}}$ the wave frequency, and $V^{\boldsymbol{k}}_{12}$ the interaction coefficients between wave modes [2]. The kinetic equation gives directly the wave dynamics on very large time scales which is hardly obtained by direct numerical simulations, and could help us to estimate energy transfers and mixing due to waves without using ad-hoc parameters [3]. Using a large scale and localized forcing, we observe that the spectrum develops through successions of nonlocal energy transfers (cf. Fig. 1), namely the Elastic Scattering (ES), Parametric Subharmonic Instability (PSI) and Induced Diffusion (ID)[4]. It leads to the formation of strongly anisotropic spectra.

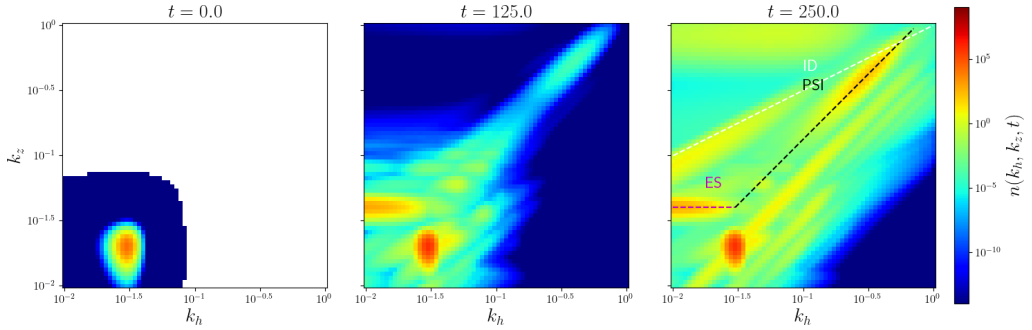

Figure 1. Wave action spectrum in one of our kinetic equation simulations at different times. k_h and k_z are the horizontal and vertical wave-vector modulus. The lines correspond to nonlocal interactions with the forced mode.

References

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