Locally varying multifractality underlies intermittent energy dissipation in turbulence

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Understanding turbulence rests delicately on the conflict between Kolmogorov's 1941 theory of non-intermittent, space-filling energy dissipation characterised by a unique scaling exponent and the overwhelming evidence to the contrary of intermittency, multiscaling and multifractality. Strangely, multifractality is not typically envisioned as a local flow property, variations in which might be clues exposing inroads into the fundamental unsolved issues of anomalous dissipation and finite time blow-up. Using a simple construction, we show that the multifractal analysis can be fundamentally extended as a tool for educing entire fields of generalized dimensions and fractal measures underlying the energy dissipation field. For instance this allows us, for the first time, to reveal the spatial variation in the correlation dimension across the flow, as seen below in Fig. 1A. By defining a suitable measure $\Phi(\mathbf{x})$ of the spatial variation of multifractality (illustrated in Fig. 1B), we show that this grows logarithmically with the extent to which the energy dissipation varies locally around $\mathbf{x}$. In other words, much of the dissipation field remains surprisingly monofractal à la Kolmogorov, while multifractality appears as small islands in this calm sea. These results suggest new ways to understand how singularities could arise and provide a fresh perspective on anomalous dissipation and intermittency. The simplicity and adaptability of our approach also holds great promise in applications ranging from climate sciences to medical data analysis.

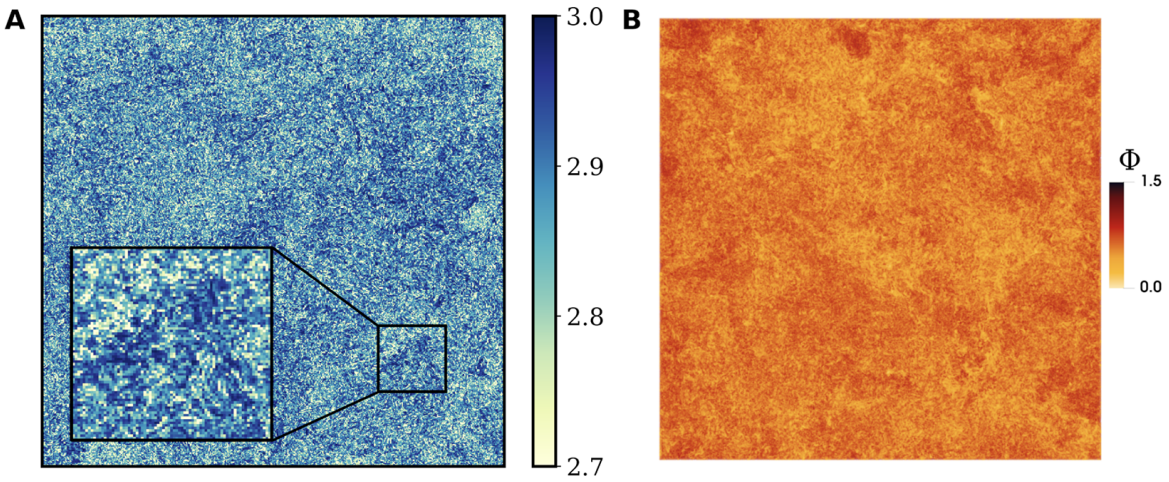

Figure 1. (A) Correlation dimension (D_2) revealed as a field using the local analysis and **(B)** Variation in the degree of multifractality, showing that turbulence is not *uniformly* multifractal.

Références

1. S. Mukherjee, S.D. Murugan, R. Mukherjee & S.S. Ray, *arXiv :2307.06074*, (2023).