Scaling laws of the plasma velocity in visco-resistive magnetohydrodynamic systems

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We consider a visco-resistive magnetohydrodynamic modeling of a steady-state incompressible tokamak plasma with a prescribed toroidal current drive, featuring constant resistivity η and viscosity ν. We reintroduce in the traditional Grad-Shafranov equation the dissipative viscous term and the non-linear $(\mathbf{v}\cdot\nabla)\mathbf{v}$ term coming from the steady-state Navier-Stokes equation [1,2,3,4,5]. It is shown that the plasma velocity root-mean-square behaves as $\eta f(H)$ as long as the inertial term remains negligible, where H stands for the Hartmann number $H \equiv (\eta\nu)^{-1/2}$, and that $f(H)$ exhibits power-law behaviours in the limits $H \ll 1$ and $H \gg 1$. In the latter limit, we establish that $f(H)$ scales as $H^{1/4}$ (cf. Fig. 1), which is consistent with numerical results [6]. These use the finite element method through the open-source platform FreeFem++ for solving partial differential equations [7].

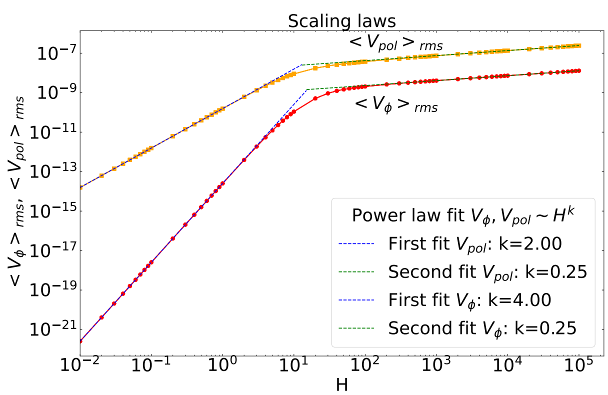

Figure 1. Root-mean square of toroidal and poloidal velocities in Alfvén velocity units as a function of the Hartmann number in log-log scale with power-law fitting curves.

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