Surface Quasi-Geostrophy : A Proxy for 3D Turbulence ?

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The Surface Quasi-Geostrophic equations (SQG), describing the two-dimensional transport of a scalar temperature field θ under strong rotation and stratification, are known to share formal similarities with the incompressible three-dimensional (3D) Navier-Stokes (NS) system [1,3]. The analogy is formally mediated by the scalar gradient, which undergoes stretching $\nabla\theta \cdot \nabla\mathbf{u}$ along Lagrangian trajectories, akin to the vortex stretching $\boldsymbol{\omega} \cdot \nabla\mathbf{u}$ constitutive of 3D-NS. Recent studies [2] have also pointed out that, when subject to suitable large-scale forcing and small-scale dissipation, SQG settles into a statistical steady state displaying hallmark signatures of canonical 3D homogeneous isotropic turbulence.

In essence, the SQG steady-state has finite energy ; It is sustained by finite injection and (anomalous) dissipation $\epsilon > 0$ through a direct cascade of kinetic energy associated with an approximate $k^{-5/3}$ power-law scaling, as prescribed by the classical Kolmogorov 1941 theory of turbulence. Here, we present a new series of highly resolved direct numerical simulations of SQG, employing up to $16,384^2$ grid points on a doubly-periodic domain. Our purpose is to expand upon previous observations, and compile the turbulent signatures observed in the forced-dissipated SQG system into a refined statistical phenomenology of SQG turbulence.

On the one hand, our numerics substantiate quantitative statistical similarities between SQG scalars and 3D-NS velocity fields. These include dissipative anomaly, negatively skewed distributions, spatial multiscaling and refined self-similarity. As dissipation vanishes, those features are observed over an increasing (inertial) range of scales delimited on the ultraviolet side by the Taylor scale λ. We argue that λ characterizes the typical size of vortex patches, originating from temperature filaments breaking up into smaller scales due to internal shearing by the underlying turbulent flow.

On the other hand, our work highlights three caveats originating from the specifics of the SQG dynamics, in particular its two-dimensionnal setting and inviscid conservation laws. *(i)* The analogy between SQG and NS holds, strictly speaking, at the level of the advected SQG scalar field only. While both the scalar and the velocity fields are multifractal, only the SQG scalar field is negatively skewed. *(ii)* Effects of finite dissipation in SQG are more pronounced than in 3D-NS. *(iii)* SQG turbulence appears less universal than 3D-NS turbulence, with likely dependence upon injection scheme.

As such, while framed in a 2D setting, SQG shares the essential statistical signatures of 3D turbulence, but, unfortunately, also its numerical challenges.

Références

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