Low-cost realization of a quantitative chaotic waterwheel

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In the 60's, several teams of researchers try to tackle the problem of better understanding thermal convection in fluids. Among them E. N. Lortenz, who was studying atmospheric convection. Using a reduction of the equations obtained while studying the Rayleigh–Bénard system, he obtained the now famous Lorenz system of three equations, which he first discovered to be able to exhibit chaotic behaviour for certain ranges of parameters [1].

Several years later, Keller and Welander, interested in oceanic convection, studied a simplified system consisting in a single rectangular fluid loop heated at its top and cooled from below [2,3].

Howard and Malkus later modified this idea by using a circular loop [4]. They discovered that its behaviour was ruled by system of equations that Lorenz had found earlier. Malkus then had the idea to build a mechanical analag to the fluid loop, where buckets losing and gaining water represented fluid being heated or cooled.

The waterwheel indeed realises a mechanical equivalent of a single convection loop, and the equation ruling its behaviour are remarkably simple :

$$\begin{aligned} \dot{\omega}/\sigma &= x - \omega \\ \dot{x} &= -x + \omega y \\ \dot{y} &= -y - \omega x + \rho \end{aligned}$$

with ω being the rotational speed of the wheel, and x, y the coordinates of its center of gravity. The behaviour of the whole system depends on only two dimentionless parameters, the Rayleigh number ρ which transcripts the competition between the weight torque and the dissipation, and the Prandtl number σ which compares two modalities of dissipation (mechanical friction and leakage).

The wheel is an excellent demonstration tool to discuss with the general public about topics such as atmospheric convection, sensitivity to initial conditions, and the difference between weather and climate. It can also be used as a tool when teaching students about dynamical systems, nonlinear physics and chaos theory.

We designed a portable, low-cost and easy to use chaotic waterwheel using 3D printing and cheap microcontrollers. A rotary encoder is used to make the wheel quantitative, allowing for real-time observation of the dynamics.

Références

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