Scaling laws of the plasma velocity in visco-resistive magnetohydrodynamic systems

Laboratoire de Physique des Plasmas (LPP), Ecole polytechnique, Palaiseau, France

Anna Krupka, Marie-Christine Firpo

The stationary Navier-Stokes equation:

$$(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \boldsymbol{J} \times \boldsymbol{B} - \nabla p + \nu \nabla^2 \boldsymbol{v}$$

Needs to be solved on the cross-section plasma domain with boundary conditions

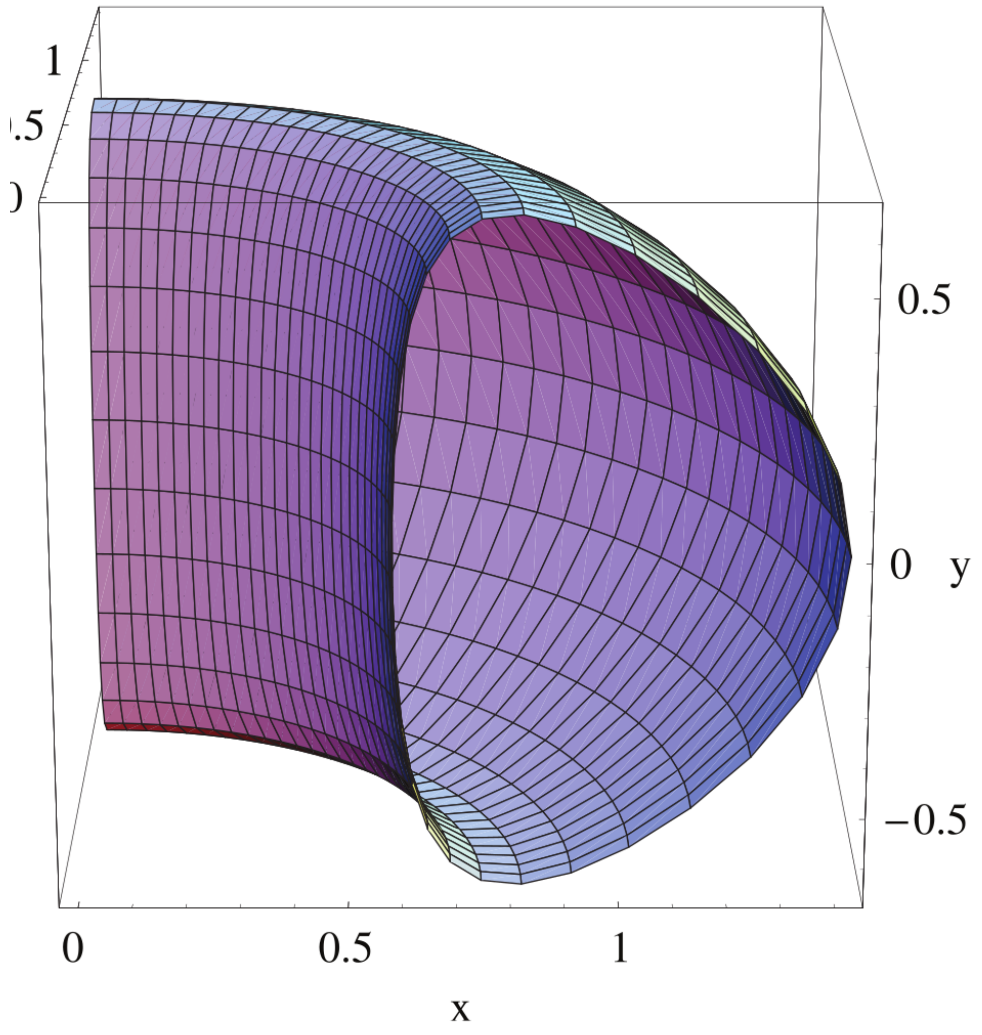
$$\nabla \cdot \boldsymbol{v} = 0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{B} = \boldsymbol{J}$$

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{J}$$



FREEFEM

[1] F. Hecht, *New development in FreeFem++*, J. Numer. Math. 20 (3-4), 251-265 (2012).

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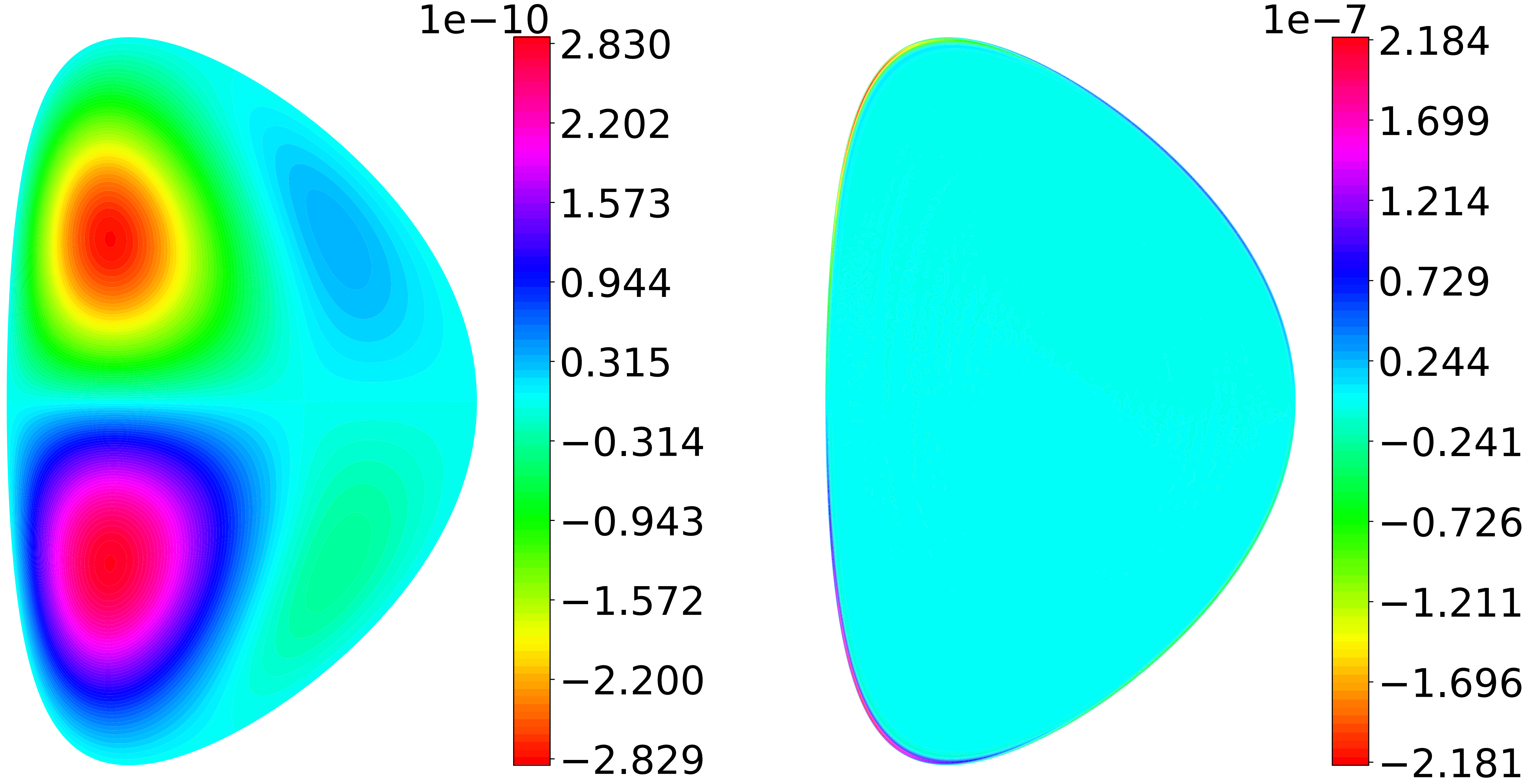

Figure 1: Toroidal velocity field for $H = 10$ and $H = 10^5$ with no-slip boundary condition for toroidal velocity with $\eta = 6.9e-9$, where is a Hartmann number $H = (\eta\nu)^{-1/2}$.

The scaling of velocity in the first regime $(H \ll 1)$ was predicted analytically [2]. The toroidal velocity in this limit scales with H^4 while the poloidal velocity scales with H^2.

[2] L. P. Kamp, D. C. Montgomery, and J. W. Bates (1998). *Toroidal flows in resistive magnetohydrodynamic steady states.*

We analytically predict the velocity behaviour in $H \gg 1$ regime by considering the boundary layer equations [3].

$$\delta \sim \frac{1}{\sqrt{H}} \qquad < v_{pol,toro} >_{rms} \sim \eta H^{1/4}$$

[3] A. Krupka and M.-C. Firpo (2023). *Scaling laws of the plasma velocity in visco-resistive magnetohydrodynamic systems*, hal-04459262.

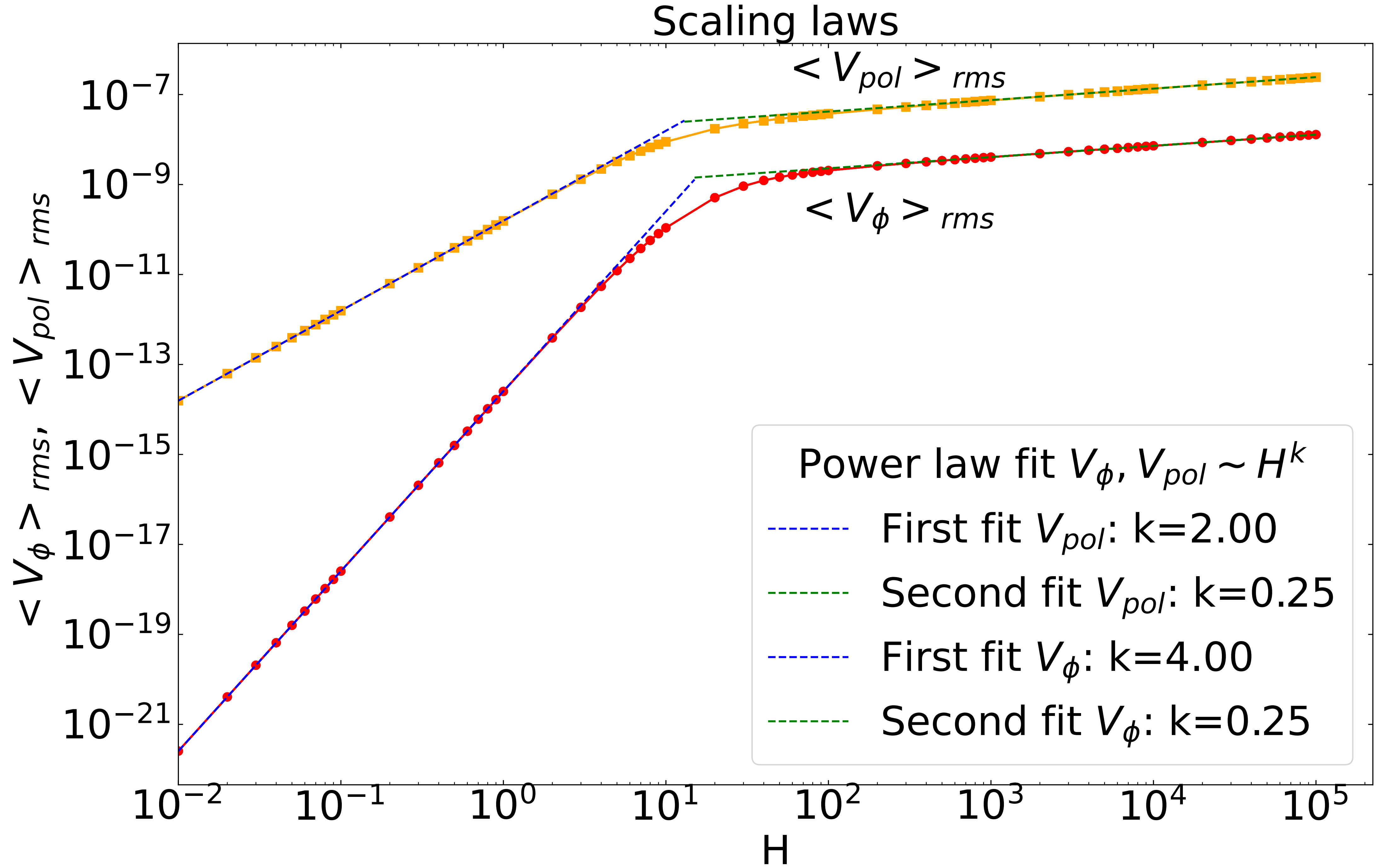

Figure 2: Root-mean square of toroidal and poloidal velocities in Alfven velocity units as a function of the Hartmann number in log-log scale with power-law fitting curves.