Reversible Navier-Stokes equation on logarithmic lattices

Costa Guillaume[1], Barral Amaury[2], Dubrulle Bérengère[3]

Université Paris-Saclay, CEA, CNRS, SPEC, 91191, Gif-sur-Yvette, France
berengere.dubrulle@cea.fr

We study the three-dimensional Reversible Navier-Stokes equations first introduced by [1], [2], in which the energy is kept constant by adjusting the viscosity over time. We perform numerical simulations of these equations using a new framework called log-lattices, to reach extremely large resolutions at a moderate numerical cost. This technique allows us to explore regimes of parameters that were out of reach of the previous direct numerical simulations of [2]. Using the non-dimensionalized forcing as a control parameter, and the square root of enstrophy as the order parameter, we confirm the existence of a second order phase transition well described by a mean field Landau theory. The log-lattices framework allows us to probe the impact of the resolution, highlighting an imperfect transition at small resolutions with exponents differing from the mean field predictions. Our findings are in qualitative agreement with predictions of a 1D non-linear diffusive model, the reversible Leith model of turbulence.

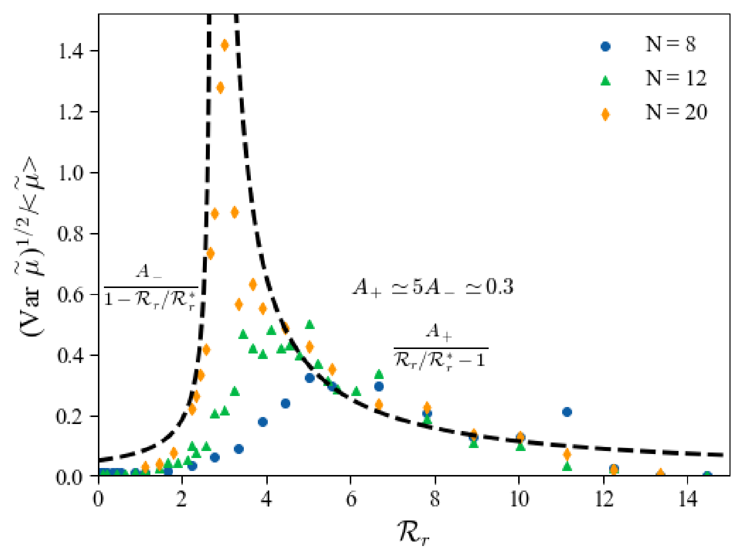

Figure 1. Second order transition for Reversible Navier-Stokes on log-lattices. Figure shows the variation of the square-root of the enstrophy as a function of a dimensionless control parameter $\mathcal{R}_r = \frac{f_0}{E_0 k_f}$.

References

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2. V.Shukla, B.Dubrulle, S.Nazarenko, G.Krstulovic & S.Thalabard, DPhase transition in time-reversible Navier-Stokes equations, *Physics Review E*, **100**, 043104 (2019).