Recent progress on smooth self-similar solutions to the compressible Euler equations

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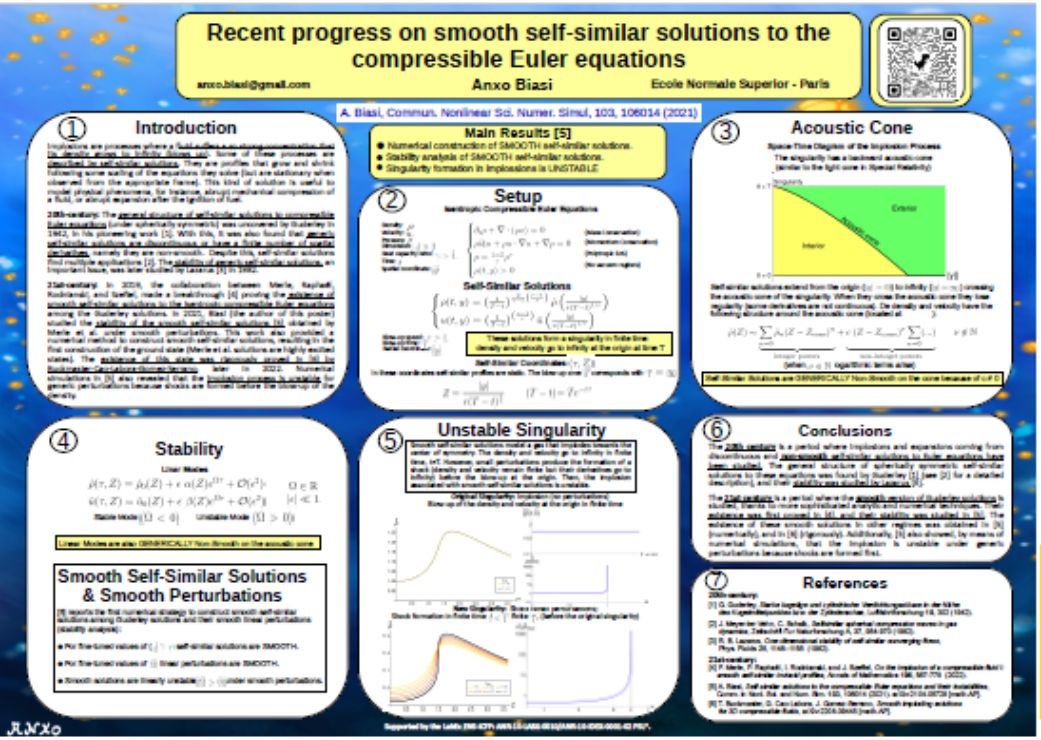

Isentropic Compressible Euler Equations

$$\partial_t \rho + \nabla \cdot (\rho u) = 0 \quad \text{(Mass)}$$

$$\rho \partial_t u + \rho u \cdot \nabla u + \nabla p = 0 \quad \text{(Momentum)}$$

$$p \propto \rho^{\gamma} \quad \text{(EoS)}$$

Singularities

Implosions: $\rho, |u| \to \infty$

Shocks: $|\nabla \rho|, \ |\nabla u| \to \infty, \quad |\rho|, |u| < \infty$

Implosions ⇨ Self-Similar Solutions

$$\rho(t,x) = \frac{1}{(T-t)^{\alpha}} \hat{\rho}\left(\frac{x}{(T-t)^{\beta}}\right)$$

Smooth Self-Similar Solutions

20th century: Non-Smooth

Guderley [1942]
Taylor [1950]
Sedov [1959]
Lazarus [1984]

21th century: Smooth

Merle et al [2019]
Biasi [2021]
Buckmaster et al [2022]

20th-century

- **Guderley [1942]:** Radially symmetric self-similar solutions: non-smooth (e.g. shocks).
- **Lazarus [1984]:** Stability of non-smooth Guderley solutions

21st-century

- **Merle-Raphael-Rodnianski-Szeftel [2019]:** Existence of smooth Guderley solutions.
- **Biasi [2021]:**
  - New smooth Guderley solutions (numerical).
  - Stability smooth Guderley solutions.
  - Instability of implosions (shock formation).
- **Buckmaster, Cao Labora, Gómez Serrano [2022]:** New smooth Guderley solutions (rigorous).