Reversible Navier-Stokes on log-lattices

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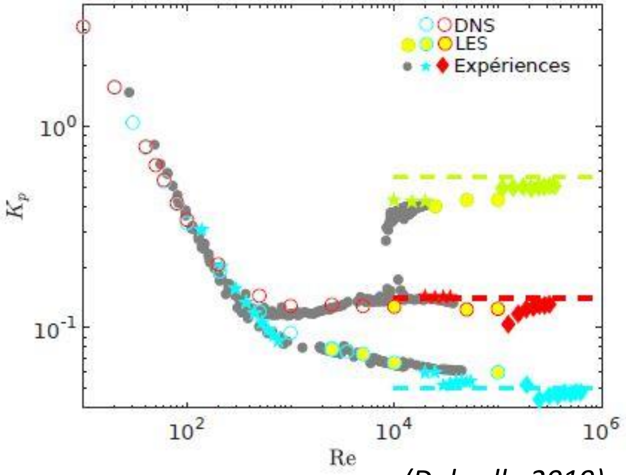
Fluid dynamics governed by Navier-Stokes equation:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \boldsymbol{\Delta} \boldsymbol{u} + \boldsymbol{f}$$

Non-linear

Viscous dissipation

In the turbulent limit (vanishing viscosity): still dissipation

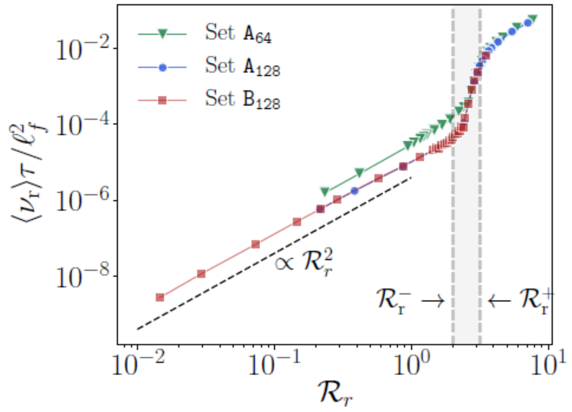


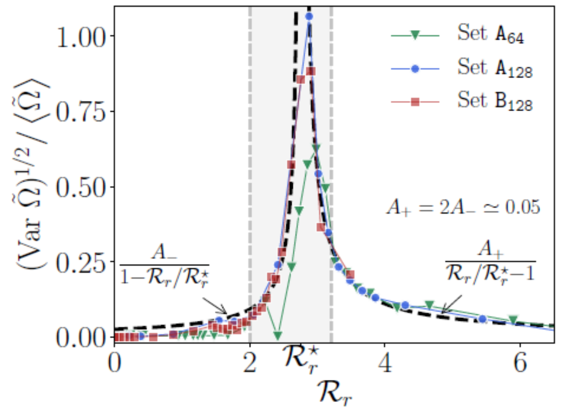
(Dubrulle 2019)

spontaneous breaking of time reversal symmetry

To study this **spontaneous symmetry breaking** we want an equilibrium system:

Introduce a **time-dependent viscosity** that restores the symmetry *(Gallavotti 1996)*





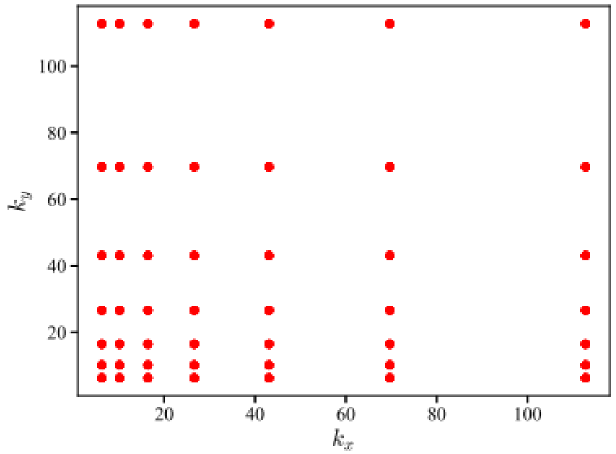
(Shukla et.al 2019)

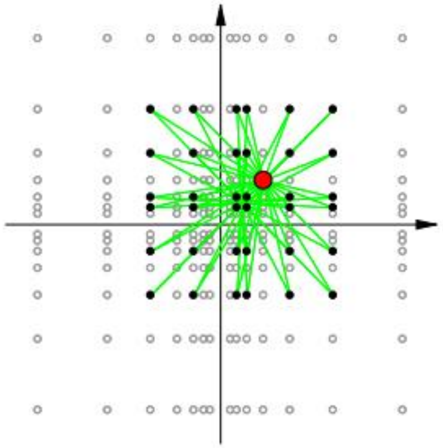
$$\mathcal{R}_\mathrm{r} = \frac{f_0}{E_0 k_f}$$

$$\nu_r = \frac{\int_\mathcal{D} \boldsymbol{f}\cdot\boldsymbol{u} d\boldsymbol{x}}{\int_\mathcal{D} \|\boldsymbol{\nabla}\wedge\boldsymbol{u}\|_2^2 \, d\boldsymbol{x}}.$$

Shukla et.al: Direct Numerical Simulations → High complexity O(Re^3), small grids

Our proposition: Log-lattices





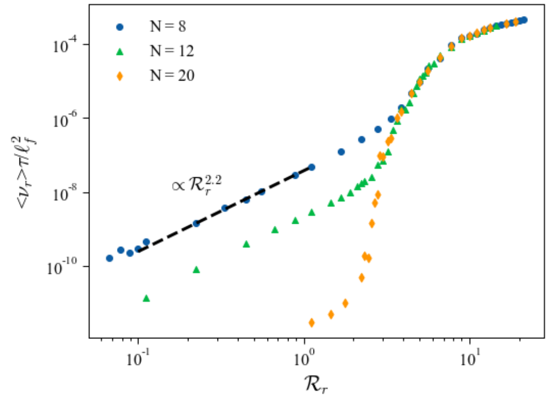
(Campolina & Mailybaev 2021)

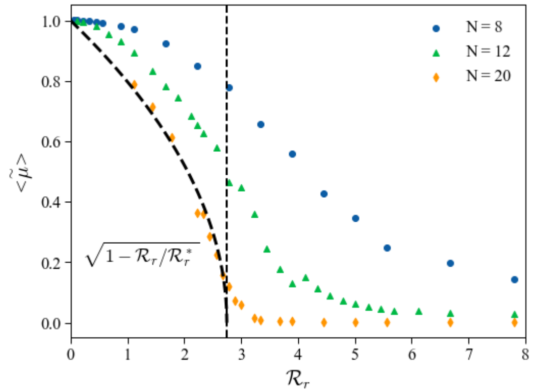
$$k = k_0 \lambda^n$$

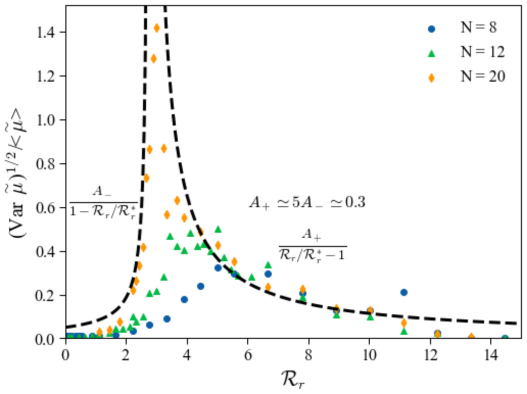
$$\lambda^m = \lambda^n + \lambda^q,\ (\mathrm{m,n,q}) \in \mathbb{Z}^3$$

Allows us to perform simulations with $k > 10^5$ on one core for < 4 days

Results:







$$\tilde{\mu} = \sqrt{\tilde{\Omega}}$$

$$\Omega(t) = \sum_{k=k_{min}}^{k_{max}} k^2 E(k,t)$$

Second order phase transition with mean field exponents for μ !

Much more to discuss thanks to log-lattices !