An efficient training method to learn a model of turbulence

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$$egin{aligned} & \left(rac{d}{dt} +
u k_1^2
ight) u_1 = i k_1 G_1[u] + f_1 & (0) \ & \left(rac{d}{dt} +
u k_2^2
ight) u_2 = i k_2 G_2[u] + f_2 & (0) \ & \left(rac{d}{dt} +
u k_3^2
ight) u_3 = i k_3 G_3[u] + f_3 & (0) \ & dots &$$



- Turbulence → expensive to simulate
- Shell Model → Simplify model of Turbulence

Use a Machine Learning Model (Long Short-Term Memory) approach to learn the Shell Model



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STD:

$$\hat{u}_{t+1} = ext{LSTM}(u_t; h_t, c_t) \ Loss(\hat{u}_{t+1}, u_t)$$

 $\hat{u}_{t+1} = \mathrm{LSTM}(u_t; h_t, c_t) ext{ if } t < h$

FOR: $\hat{u}_{t+1} = ext{LSTM}(\hat{u}_{t+1}; h_t, c_t) ext{ if } t \in [h, h+f]$

$$\mathcal{L} = \sum_{t=h}^f Loss(\hat{u}_t, u_t)$$

- Discrepance between generating and training
- Develop Forecast Training algorithm



RESULTS



Energy spectrum for each type of training, the model trained with forecast training shows an improvement in performance.

In the figure, we show the Squared Error between the anomalous scaling exponents evaluated with the trajectories generated by the model trained with the forecast method and by the model trained with the standard training. We show a performance improvement