Spirographic motion in a vortex

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Studies of particle motion in vortical flows have mainly focused on point-like particles, either inertial or self-propelled. This approximation assumes that the velocity field that surrounds the particle is linear. We consider an inertialess rigid dumbbell in a two-dimensional steady vortex [Fig. 1(a)]. While this system remains analytically tractable, the particle experiences the nonlinearity of the surrounding velocity field. By exploiting the rotational symmetry of the flow, we reduce the problem to that of a two-dimensional dynamical system, whose fixed points and periodic orbits can be used to explain the motion of the dumbbell. In particular, if ℓ is the length of the dumbbell, r_c the radial distance of its center of mass, and α its orientation angle, we show that the quantity $(r_c/\ell)\exp(-2r_c^2/\ell^2)\cos\alpha$ is a constant of motion irrespective of the form of the vortex. This result has different implications depending on the variation of the fluid angular velocity with the radial distance. For all vortices in which the fluid angular velocity decreases with the radial distance, the dynamics is qualitatively the same and consists of a spirographic quasiperiodic motion around the vortex center. This results from a periodic oscillation in the radial direction combined with a revolution around the center. The shape of the trajectory depends strongly on the initial position and orientation of the dumbbell (see Fig. 1(b) for a representative trajectory of the center of mass of the dumbbell in the Lamb–Oseen vortex). If the fluid angular velocity is not monotonic, the spirographic motion is altered by the existence of transport barriers, whose shape is now sensitive to the details of the vortex.

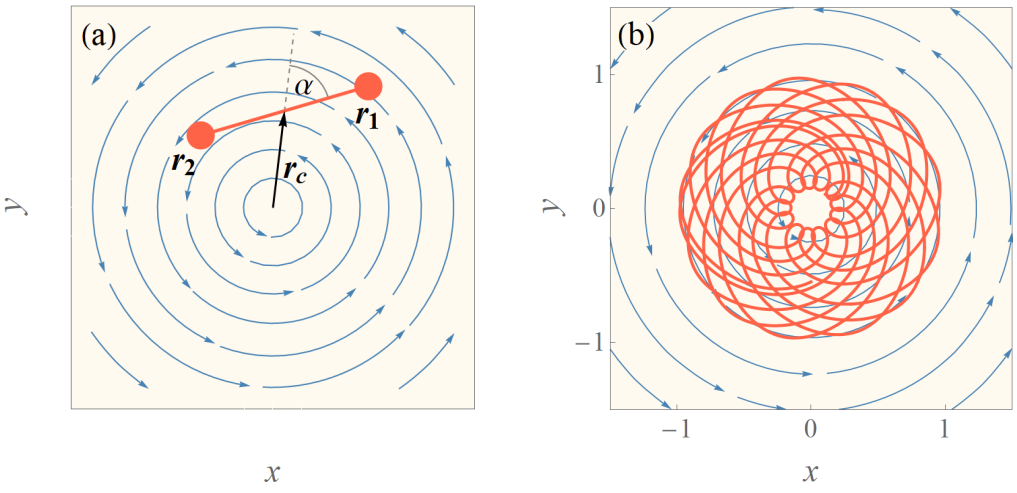

Figure 1. (a) Schematic of the dumbbell in a vortex. (b) A representative trajectory of the center of mass of the dumbbell in the Lamb–Oseen vortex.

Références

1. S.R.Yerasi, R.Govindarajan & D.Vincenzi, Spirographic motion in a vortex, arXiv:2201.05448 (2022).