Diffusion transitions in a 2D periodic lattice

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Spatial diffusion of particles in periodic potential models has provided a good framework for studying the role of chaos in global properties of classical systems. Here a bidimensional "soft" billiard (fig 1, left), which is a classical dynamics derived from an optical lattice hamiltonian system [1], is used to study diffusion transitions under variation of the control parameters, namely the total energy and a lattice coupling parameter [2].

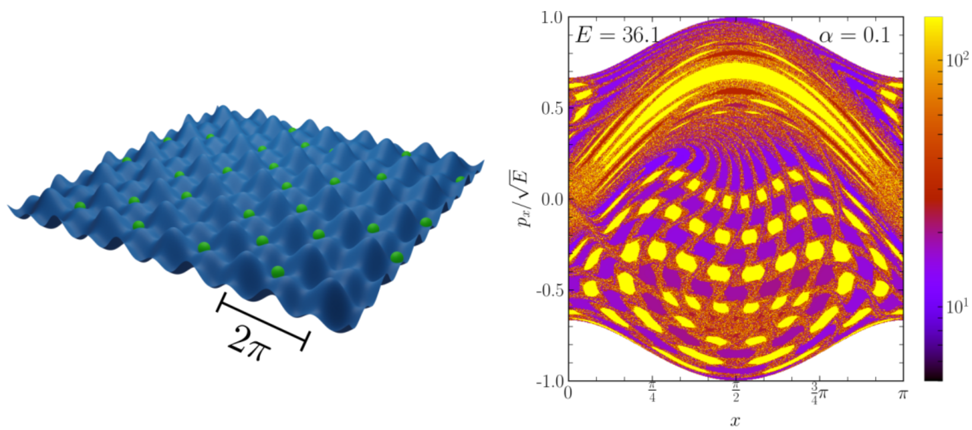

Figure 1. (Left) Soft billiard potential periodic surface. (Right) Island myriad bifurcation phenomenon observed in a Poincaré Surface Section.

Sudden transitions in diffusion between normal and ballistic regimes are found and characterized by inspection of topological changes in phase-space. Particular transitions correlated with increases in global stability area are shown to occur for energy levels where local maxima points become accessible, deviating trajectories approaching them. Even though these points are unstable, they promote the emergence of regular structures in phase-space with a slowing down of the dynamics and an island myriad bifurcation phenomenon, along with the suppression of long flights within the lattice. At transition, phase-space becomes populated by a web-like manifold structure, with multiple isochronous chains of even discrete period (fig 1, right). These chains form layers with increasing period and alternate displacement range in the lattice due to stable periodic orbits with close or open spatial periodic topology. Other diffusion regime variations occurring within small intervals of control parameters are shown to be related to the emergence of a set of orbits with long flights, thus altering the total average displacement for long integration times but without global changes in phase-space.

References

1. E. Horsley, S. Koppell & L. E. Reichl, Chaotic dynamics in a two-dimensional optical lattice, *Phys. Rev. E*, **89**, 012917 (2014).
2. M. Lazarotto, I. Caldas & Y. Elskens, Diffusion transitions in a 2D periodic lattice, *arXiv:2111.04450*, (2021).