A Dynamical Model of the Turbulent Energy Cascade

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In three-dimensional Navier-Stokes turbulence, energy is injected at a large scale L and is efficiently transported to small scales. In this process, the fluid reaches a state of finite variance and large spatial gradients, which can be approximately described by a rough velocity field of Hölder exponent $H \approx 1/3$. Motivated by this phenomenon, in two recent works [1,2], we have studied a stochastic partial differential equation (SPDE) for a complex velocity field u in one spatial dimension that is randomly stirred by a spatially smooth and uncorrelated in time forcing term. This dynamics includes linear operators responsible for a cascading transfer of energy from large to small scales [3,4] and to the development of fractional regularity of order H. Multifractal corrections are included drawing inspiration from a known probabilistic construction, the Gaussian multiplicative chaos, which motivates a quadratic nonlinear interaction in this model. Through numerical simulations, we observe the non-Gaussian and in particular skewed nature of these solutions, as can be seen in Fig. 1, an important feature in the modeling of turbulent velocity fields.

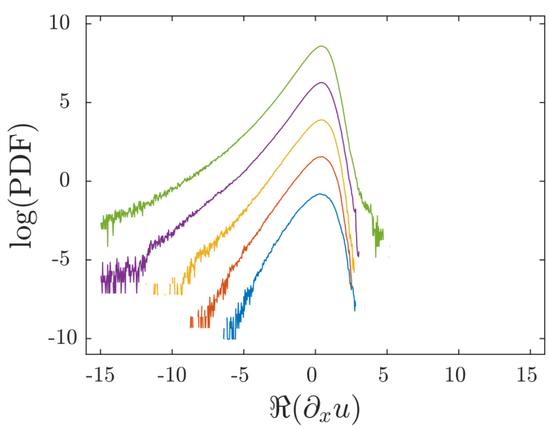

Figure 1. Histogram of the real part of the velocity gradient $\Re\partial_x u_{H,\gamma,\nu}$. Different colors represent different viscosities (or Reynolds numbers), where the uppermost curve corresponds to the smallest viscosity. The curves have been arbitrarily shifted for clarity.

Références

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