Regularization theory of singular bifurcations

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Bifurcation theory has become an inevitable branch of research across disciplines. Nonlinearities make, in general, systems with an explicit exact solution to be an exception. The classical method is then to develop a perturbation expansion, expressing the solution as a regular power series in terms of an amplitude describing the bifurcation. However, the existence of exact nonlinear solutions, even if rare, revealed recently that this analytical expansion can be defective, in that the bifurcation equation is singular (cannot be expanded in an analytic power series) leading to novel types of bifurcations, called here singular bifurcations.

We will first illustrate our investigation with an exactly solvable model of microswimmers exhibiting a singular bifurcation from the stationary state to motility as a function of activity. We then propose an original theory which allows us to show how singular bifurcations can emerge, and how to properly treat them. Furthermore, we provide a systematic classification of singular bifurcations.

We show in this study that a singular bifurcation can be regularized to a classical bifurcation by a small perturbation of the singular problem. The regularized problem shows the classical form of the perturbative solution but this solution agrees with the exact solution of the regularized problem only in a small range of perturbation amplitudes and does not, in general, have anything in common with the singular solution for any finite perturbation amplitude. The coefficients of the perturbative expansion diverge as the regularization parameter tends to zero.

We propose a method of analytic extrapolation that allows us to recover correctly the singular solution from the truncated perturbative expansion of the regularized problem. This method allows us to reconstruct quantitatively the singular limit and the solution of the regularized problem for perturbation amplitudes beyond the radius of convergence[1].

Références

1. A. Farutin and C. Misbah, *arXiv :2112.12094*, (2022)