Low frequency spectra of bending wave turbulence

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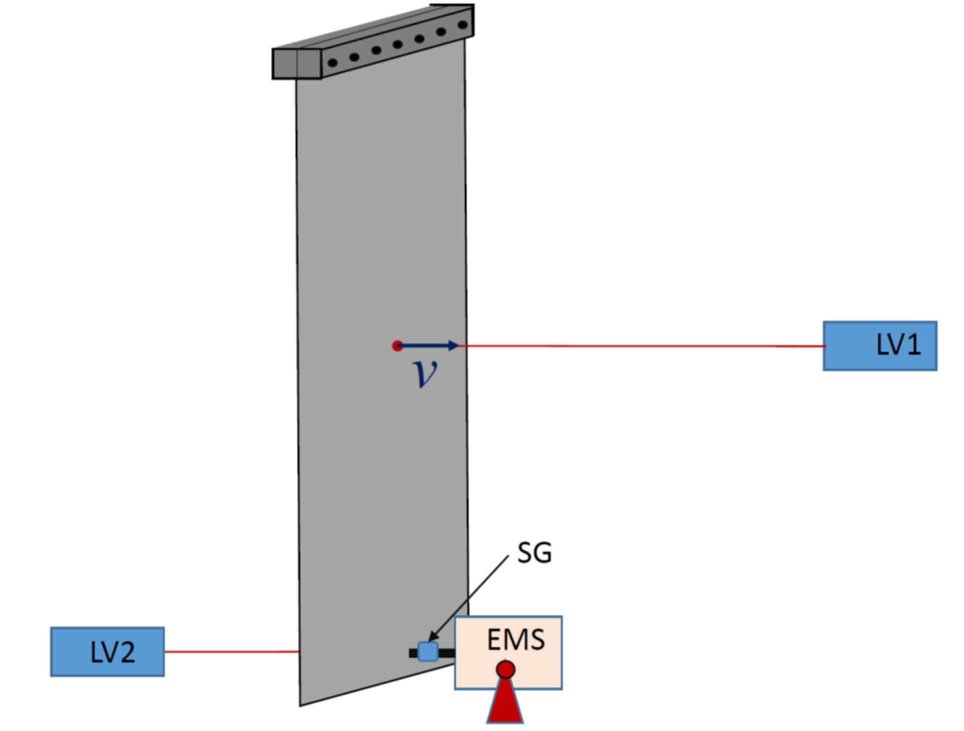
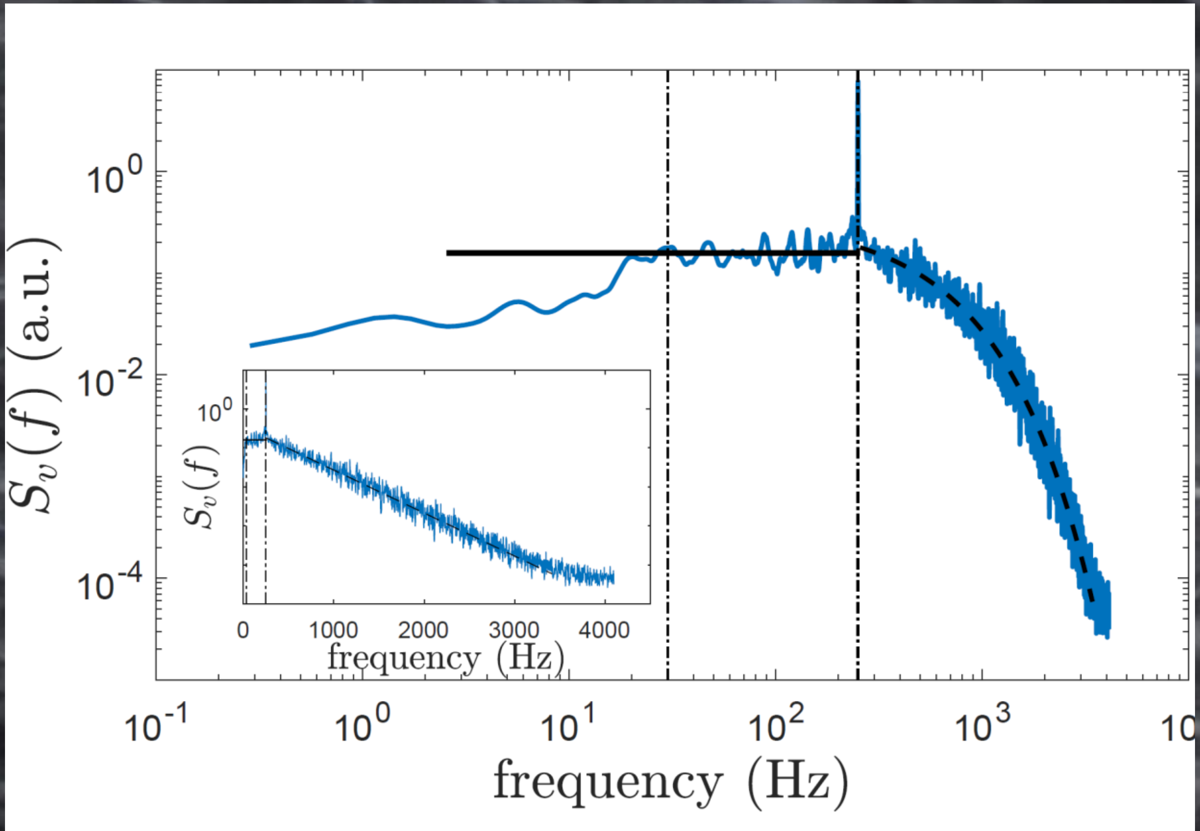
Motivations: Study of the large sale\low frequency spectra in wave turbulent systems without inverse cascade

- Do the frequency spectra agree with an equipartition of energy in this out-of-equilibrium system?
- Can we infer the spectrum at low frequency form the driving parameters**?**

Bending waves experimental systems



Thin stainless steel plate
(H=2mXL=1mXh=0.5mm)



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Motivation :

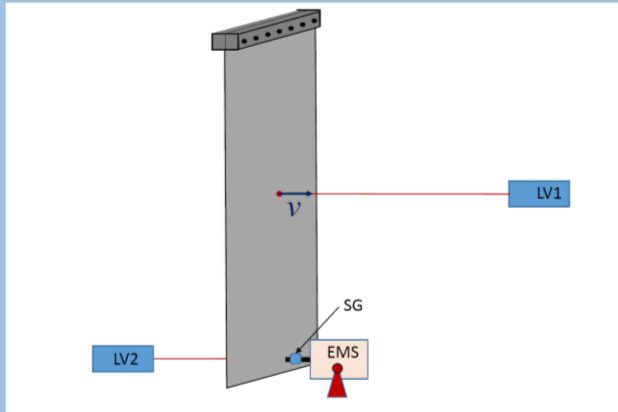
Most of the studies on turbulence focus on the direct cascade of energy form large energy injection scale to small dissipative scale. However the properties of the scales larger than the forcing are also relevant in many phenomena where they are involved. In 3D turbulence the equipartition predicted for this large scale has been evidenced only recently.

For wave turbulence, an inverse cascade may drive the large scale behaviors, but we consider here on the bending wave turbulence where no fluxes are expected through the largest scales. We focus on the following issues:

- Does the low frequency part of the spectra of vibrated plates agree with an equipartition of energy in this out-of-equilibrium system?
- Can we infer the spectrum at low frequency form the driving parameters**?**

Experimental devices

A thin stainless steel plate (H=2mXL=1mXh=0.5mm) is forced by a Electromagnetic Shaker (EMS). The perpendicular velocity v is measured in the middle of the plate with a Laser Vibrometer (LV1). The force and the velocity at the injection point are also measured to estimate the injected power I.



Dispersion relation of the bending waves: $2\pi f = chk^2$

Results

Power Density Spectra $S_v(f)$

- $f \le f_c = 30$ Hz plate's eigenmodes
- $f_c < f \le f_o$; $S_v^<(f) = Cte$
- $f > f_o$; $S_v^>(f) = C \cdot \exp(-\tau_o f)$

With f_o the forcing frequency

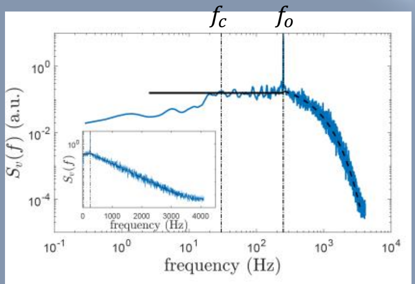


FIG. 2. PDS of the perpendicular velocity measured at a point in the middle of the plate with f_o = 250 Hz and an input power of 407 mW. The continuous line represents the plateau value evaluated between 30 and 220 Hz. **Main panel**: the PDS is plotted with logarithmic axes. **Inset**: semi-logarithmic axes are used. The vertical dot-dashed line point out the low frequency cutoff at f_c = 30 Hz and the forcing frequency at f_o = 250 Hz.

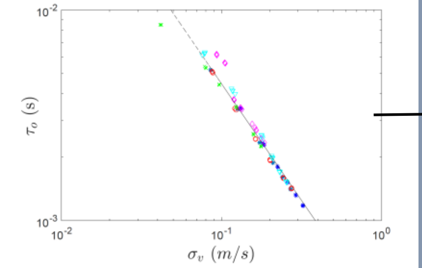


FIG. 3. The characteristic time of the exponential decay of the PDS of v, as a function of the standard deviation of v. The various forcing frequencies are: f_o = 100 Hz ∗(blue online), f_o = 150 Hz ∘ (red) , f_o = 200 Hz × (green) , f_o = 250 Hz ⋄ (magenta), f_o = 300 Hz ∇ (cyan). The dashed line represents the best fit of the experimetal data. It gives a exponent of -1.1±0.1.

$\tau_o = \alpha/\sigma_v$

α a length$\sim\mathcal{O}(h)$

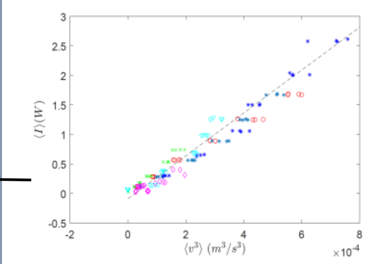


FIG. 4. The mean injected power $\langle I \rangle$ as a function of the 3rd moment of the perpendicular velocity v . Symbols are as previously: f_o = 100 Hz ∗(blue online), f_o = 150 Hz ∘ (red) , f_o = 200 Hz × (green) , f_o = 250 Hz ⋄ (magenta), f_o = 300 Hz ∇ (cyan). The dashed line represents the best linear fit wit a slope about 0.3 kg/m. The horizontal data scattering is due to the uncertainty on $\langle v^3 \rangle$ obtained from four identical experimental runs.

$\langle I \rangle \propto \epsilon \propto \langle v^3 \rangle$

0[th] law of turbulence

With ϵ the dissipation rate.

Interpretation

- Dimensional analysis:

$$\frac{S_v}{ch} = F\left(\frac{fh}{\epsilon^{\frac{1}{3}}}, \frac{f_o h}{\epsilon^{\frac{1}{3}}}, \frac{\epsilon}{c^3}\right); \ \frac{S_v}{ch} \xrightarrow[f_o \gg \epsilon^{1/3}/h]{} C\left(\frac{f_o h}{\epsilon^{\frac{1}{3}}}, \frac{\epsilon}{c^3}\right) \cdot exp\left(-\frac{fh}{\epsilon^{\frac{1}{3}}}\right)$$

- Assumption: non energy flux at large scale ⇒Equipartition for $f < f_o$ although energy exchange by nonlinearity possible.

$S_v(k) = \frac{e(k)}{L}$ with $e(k)dk = \frac{2\pi k dk}{\beta(2\pi L)^2 \rho L^2}$ and $S_v(k)dk = S_v(f)df$

$\Rightarrow \boldsymbol{S_v^<(f) = \frac{1}{2\beta\rho c L h}}$ for $f < f_o$

β^{-1} being the energy per mode

- Assuming spectrum continuity at $f = f_o$ and with $\sigma_v^2 = \int_0^\infty S_v(f)df$

$\Rightarrow C = \frac{\alpha\sigma_v^2}{\alpha f_o + \sigma_v} \exp\left(\frac{\alpha f_o}{\sigma_v}\right)$ and $\boldsymbol{\beta^{-1} = 2\rho c h L \cdot \frac{\alpha\sigma_v^2}{\alpha f_o + \sigma_v}}$

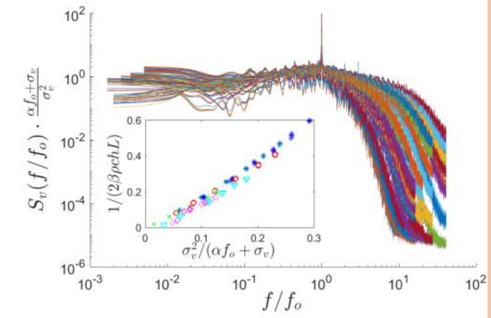


FIG. 5. PDS of v, the velocity of the perpendicular displacement of the plate, rescaled by the expected plateau value: $\sigma_v^2/(\alpha f_o + \sigma_v)$ as a function of the reduced frequencies f/f_o. Inset: linear relation between the plateau value and $\sigma_v^2/(\alpha f_o + \sigma_v)$ with the forcing frequency: f_o = 100 Hz ∗(blue online), f_o = 150 Hz ∘ (red) , f_o = 200 Hz × (green) , f_o = 250 Hz ⋄ (magenta), f_o = 300 Hz ∇ (cyan).

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