Paths to synchronization

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This work shows a new approach to the study of dynamic systems that act on a graph $\mathcal{G} = (V, E)$ and that synchronize. As a first example, we take a simple linear system, known as the Laplacian associated with the adjacency matrix of $\mathcal{G}$ the ODE on $\mathbb{R}^{|V|}$

$$\frac{d}{dt}x = L_{\mathcal{G}}(x), \tag{1}$$

where $|V| = n$ and $L_{\mathcal{G}}$ is the Laplacian matrix of the adjacency matrix of $\mathcal{G}$. Which can also be written as:

$$\frac{d}{dt}x_k = \sum_{j\in\mathcal{V}(k)} (x_j - x_k), \tag{2}$$

where x_k are the coordinates of the vector x and $\mathcal{V}(k)$ denotes the set of closest neighbors of vertex k.

This system is such that the diagonal

$$\Delta = \{x \in \mathbb{R}^n : x_i = x_j \ \forall\ 0 \le i \le j \le n\} \tag{3}$$

is a **global attractor**, that is, such that $x(t) \to \Delta$ as $t \to \infty$ for all initial condition $x \in \mathbb{R}^n$.

The second system that we analyze is the nonlinear system, known as the Kuramoto Model which is the ODE on $\mathbb{R}^{|V|}$

$$\frac{d}{dt}\varphi_k = \omega_k + \sigma \sum_{j\in\mathcal{V}(k)} \sin(\varphi_j - \varphi_k), \tag{4}$$

where $\mathcal{V}(k)$ denotes the set of closest neighbors of node k, the natural frequencies are distributed according to some probability density $\omega \mapsto g(\omega)$ and σ is the coupling strength with a suitable scale, so that the model has a good behavior when $|V| = n \to \infty$. The conditions under which we observe synchronization behaviors are well known.

We make a comparative study of both systems with the approach proposed here.

References

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