Beyond the tip of the parametric instability tongue

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Parametric instabilities are dynamical instabilities which can arise when the mechanical state of a structure is periodically modulated in time. It is often described as a phenomenon one wishes to avoid: for example the parametric rolling observed on sailing ships and is also exploited to study vibrating fluids (Faraday waves [1]) or Nano-Electro- Mechanical Systems [2]. One well-known limitation when fully exploiting classic parametric instabilities based on small periodic modulation of a mechanical state is that inherent friction forces rapidly cancel sub harmonic parametric resonances. To overcome this drawback, we propose to periodically vary the evolution function of a dynamical system to enhance and better control parametric instabilities.

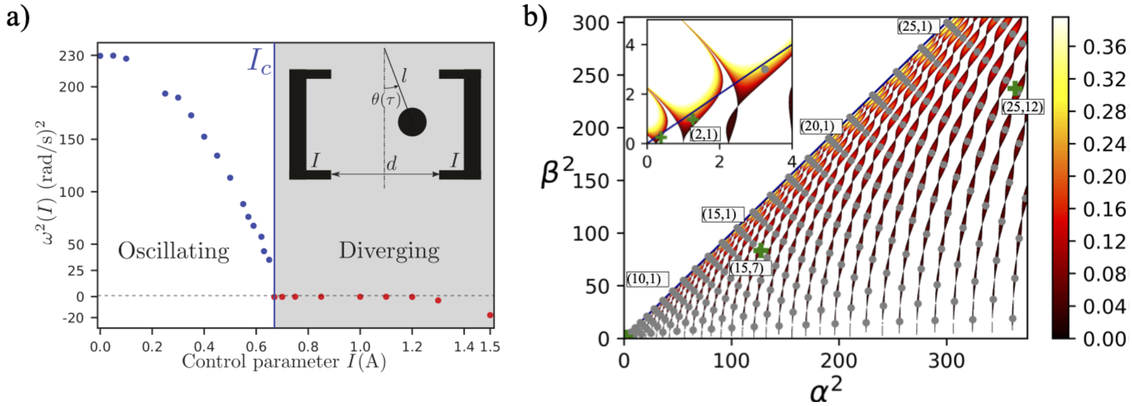

Figure 1. Triggering extremely high orders of parametric resonances. a) Experimental Floquet oscillator: pendulum with a metallic marble symmetrically placed between two identical attracting electromagnets whose force depends on the imposed electrical current I (see sketch in inset). The scalar $\omega^2(I)$, characterize the local evolution function of the electromagnetic pendulum. The electromagnets are periodically turned on and off in a square wave fashion. b) Extended stability diagram of the Meissner equation [3] showing the evolution of the Floquet exponent up to 0.4 in the parameter space. Grey dots (m, n) represent the discrete solutions using our geometrical relation. Green crosses represent our experimental results. Inset zooms on the classic first instability regions.

By combining theoretical models and desktop experiments, we have developed a geometrical relation to trigger high order parametric resonances (Grey dots in Fig 1.b)). As a proof of concept, we study the motion of an electromagnetic pendulum (Fig 1.a) and we are able to observe extremely high orders of parametric resonance which we can control, even in the presence of dissipation (Green crosses in Fig1.b)). These concepts being universal, we can then think of new promising dynamic functionalities which could be applied to dynamical systems with a natural cycle which time scale could be periodically varied.

References

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