Spontaneous oscillations of liquid bells

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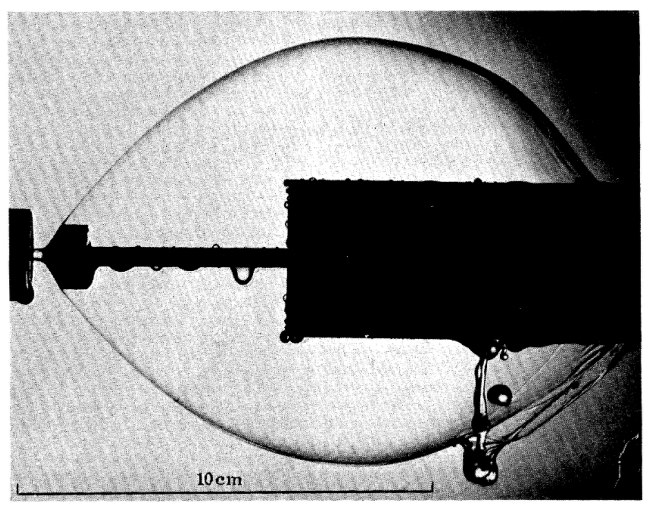
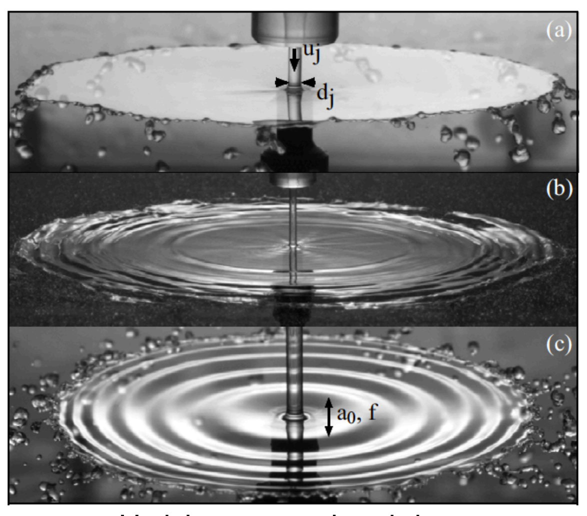
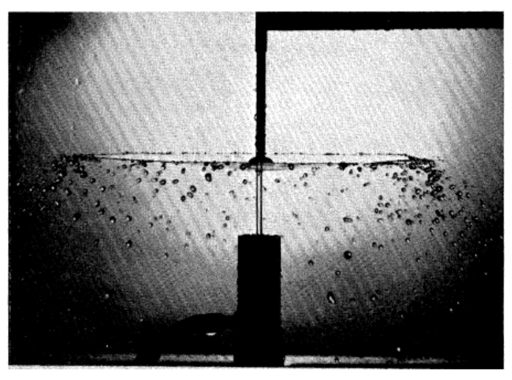
History of liquid bells from Taylor to today ...



Jet impacting on conical object
G.I. Taylor (1959)



Undulations on a liquid sheet
Bremond & Villermaux

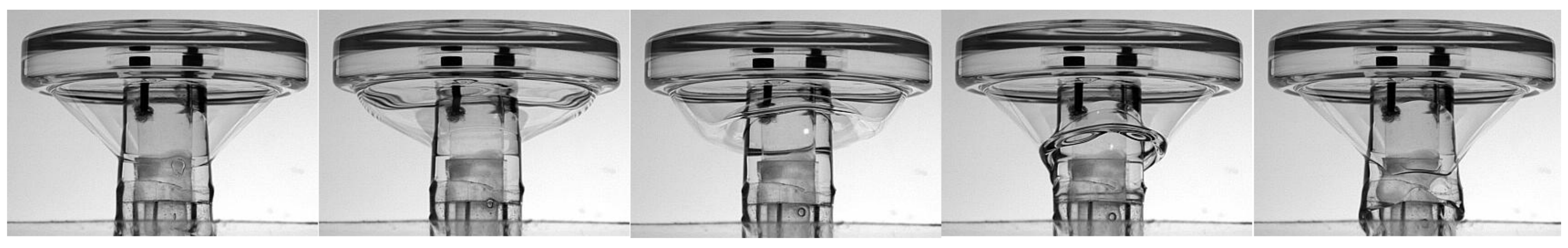


Polygonal viscous sheets
J.W. Bush (Unpublished)

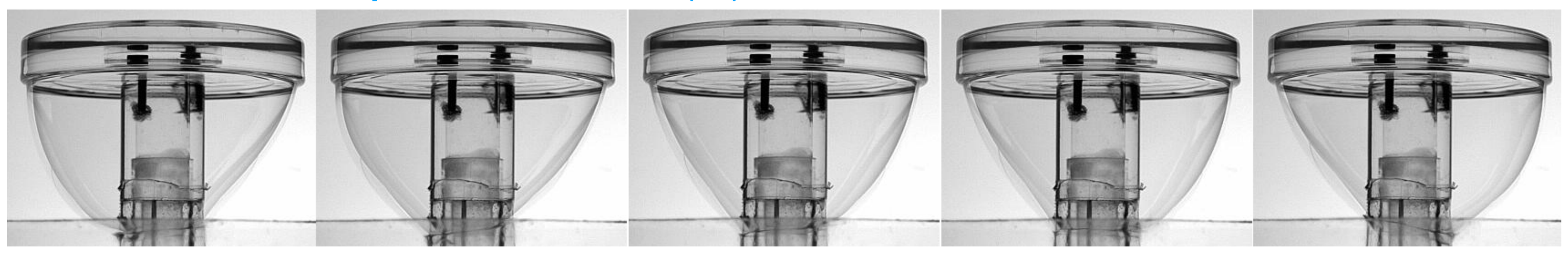
Crumpled water bells
Lhuissier and Villermaux JFM (2012)

Spontaneous oscillations

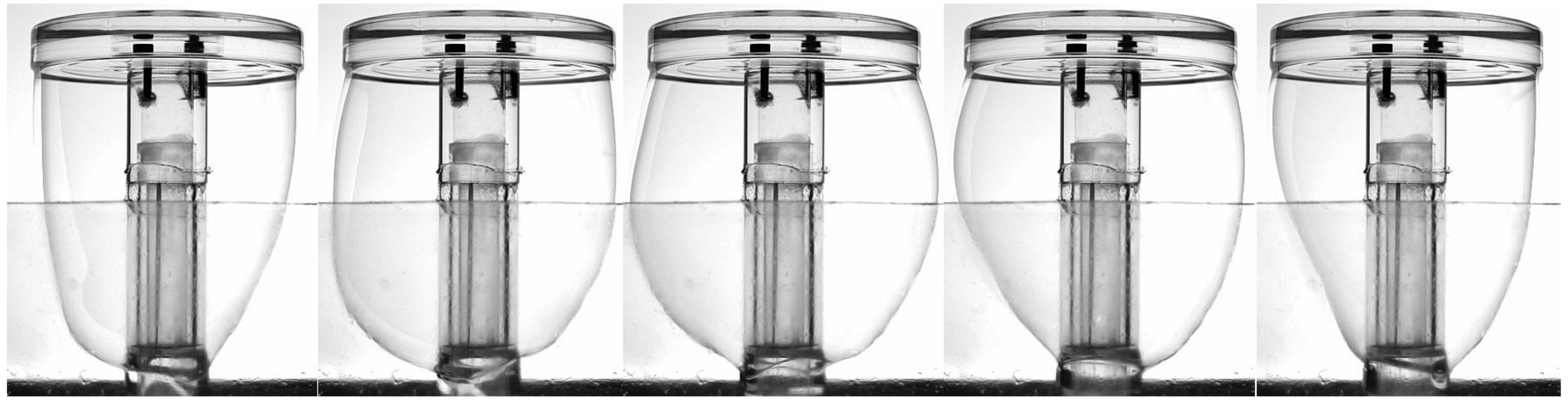
Bell oscillates at well-defined frequency f below a (volume-dependent) critical flow-rate Q_c



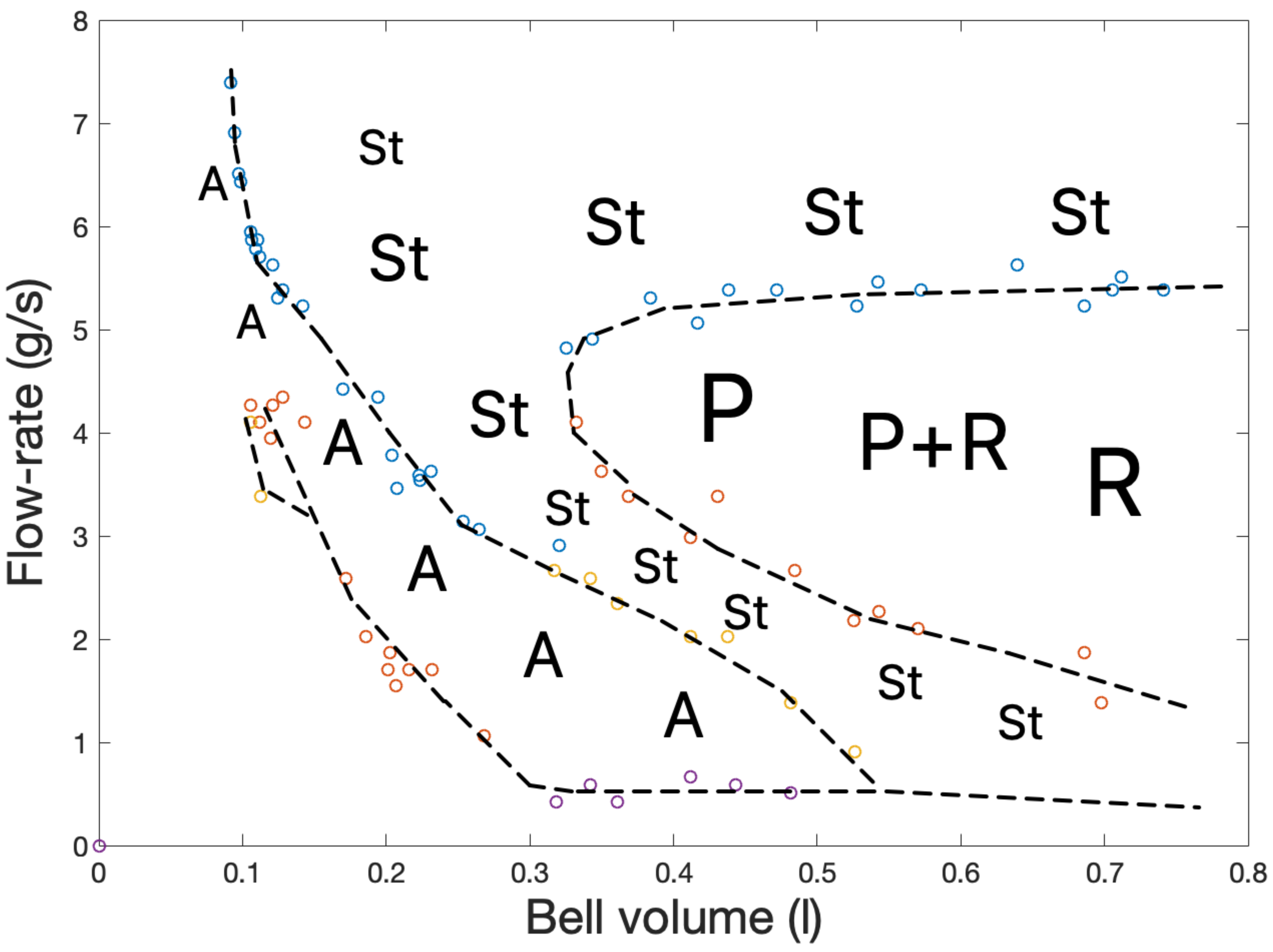
Small volume : Axisymmetric mode (**A**)



Medium volume : Planar mode (**P**)



Large volume : Rotational mode (**R**)



Conclusions - Ongoing work

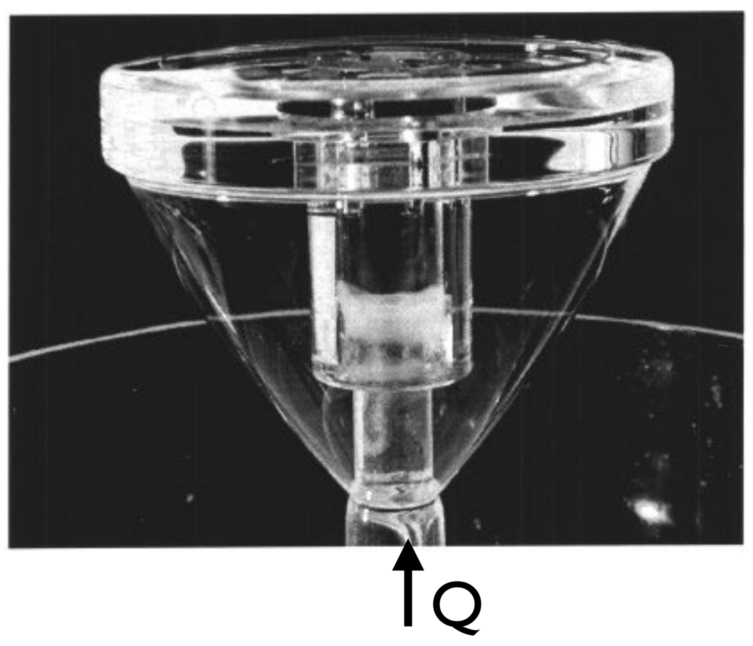
Spontaneous oscillations of liquid bells falling from are observed at low-flow rate, when We<1 everywhere in the sheet.

Three spatial modes Axisymmertric, Planar and Rotational, depending on flow-rate Q and bell volume V.

Well-defined frequency (between 3 and 5.5 Hz) and amplitude, but no obvious relationship between f and flow-rate or bell volume.

Possible coupling with air flow inside the bell ?

Exp. setup (simplified)



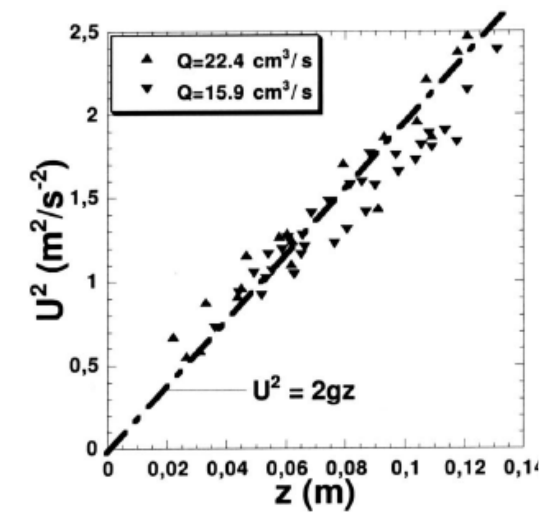
Silicon oil (100 cSt) overflows from a circular dish at constant flow-rate

$$Q = 2\pi U R h$$

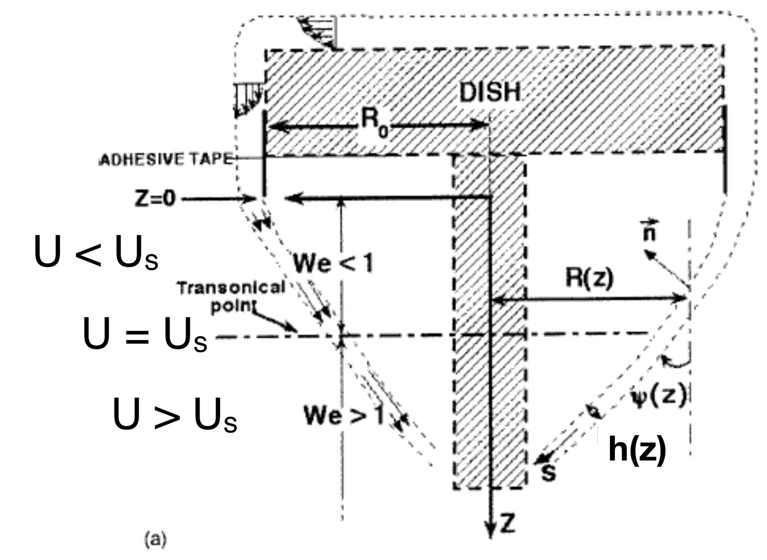
U(z) : fluid velocity
R(z) : Bell external radius
h(z) : sheet thickness

Bell volume is controlled by inflating/deflating

Flow structure of transonic bells



P.B., C. Clanet and L. Limat.
Transonic liquid bells. Phys. Fluids (2004)



Free-fall flow in the sheet

$$U^2 = U_0^2 + 2gz$$

$$U_0 = \left(\frac{\rho g}{12\mu\pi^2 R_0^2}\right)^{1/3} Q^{2/3}$$

$$We = \frac{\rho h U^2}{2\sigma} \quad \frac{\text{Inertia}}{\text{Surface tension}}$$

$U_0 \sim 1$ cm/s ⟶ **We(z) < 1 close to overhang**

We(z) increases with z

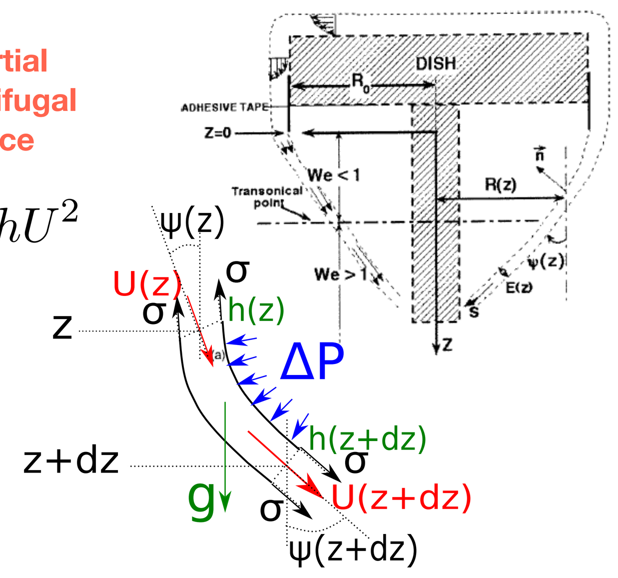
$$U_s = \left(\frac{2\sigma}{\rho h}\right)^{1/2} \qquad We = \frac{\rho h U^2}{2\sigma} = \left(\frac{U}{U_s}\right)$$

U_s : surface wave velocity (« sound wave »)

Shape determination : from analytics to numerics

Surface tension: Axisymmetric curvature, Planar curvature; Gravity; Internal pressure; Inertial centrifugal force

$$2\sigma\left[\frac{\cos\psi}{R} - \frac{d\psi}{dS}\right] + \rho g h \sin\psi = \Delta P - \frac{d\psi}{dS}\rho h U^2$$



$$\frac{d\psi}{dS}\left(\frac{\rho Q U}{4\pi\sigma} - R\right) + \cos\psi = \frac{\Delta P}{2\sigma}R - \frac{\rho g Q}{4\pi\sigma U}\sin\psi$$

$$r = \frac{R}{R_0};\ z = \frac{Z}{R_0};\ s = \frac{S}{R_0};\ u = \frac{U}{\sqrt{2gR_0}}$$

Dimensionless equation

$$(2\beta u - r)\frac{d\psi}{ds} + \cos\psi = \alpha r - \beta\frac{\sin\psi}{u}$$

Two parameters

$$\alpha = \frac{\Delta P R_0}{2\sigma} \qquad \beta = \frac{\rho Q\sqrt{g}}{4\pi\sigma\sqrt{2R_0}}$$

Analytical expression (weak curvature and ΔP=0)

$$r(z) = 1 - \frac{2u_0^3}{3\beta}\left[\left(1 + \frac{z}{u_0^2}\right)^{3/2} - 1\right]$$

Quantitative results (frequency, amplitude)