Counterflow-induced inverse energy cascade in three-dimensional superfluid turbulence

Juan Ignacio Polanco   Giorgio Krstulovic

Université Côte d'Azur, Observatoire de la Côte d'Azur, CNRS, Laboratoire Lagrange, Nice

Phys. Rev. Lett. **125**, 254504 (2020)

Superfluidity of liquid helium

- Helium-4 displays **superfluidity** below $T_\lambda \approx 2.17$ K
- At $1 \lesssim T < T_\lambda$: **two-fluid phenomenology**
  → coupling between normal (viscous) and superfluid (inviscid) components

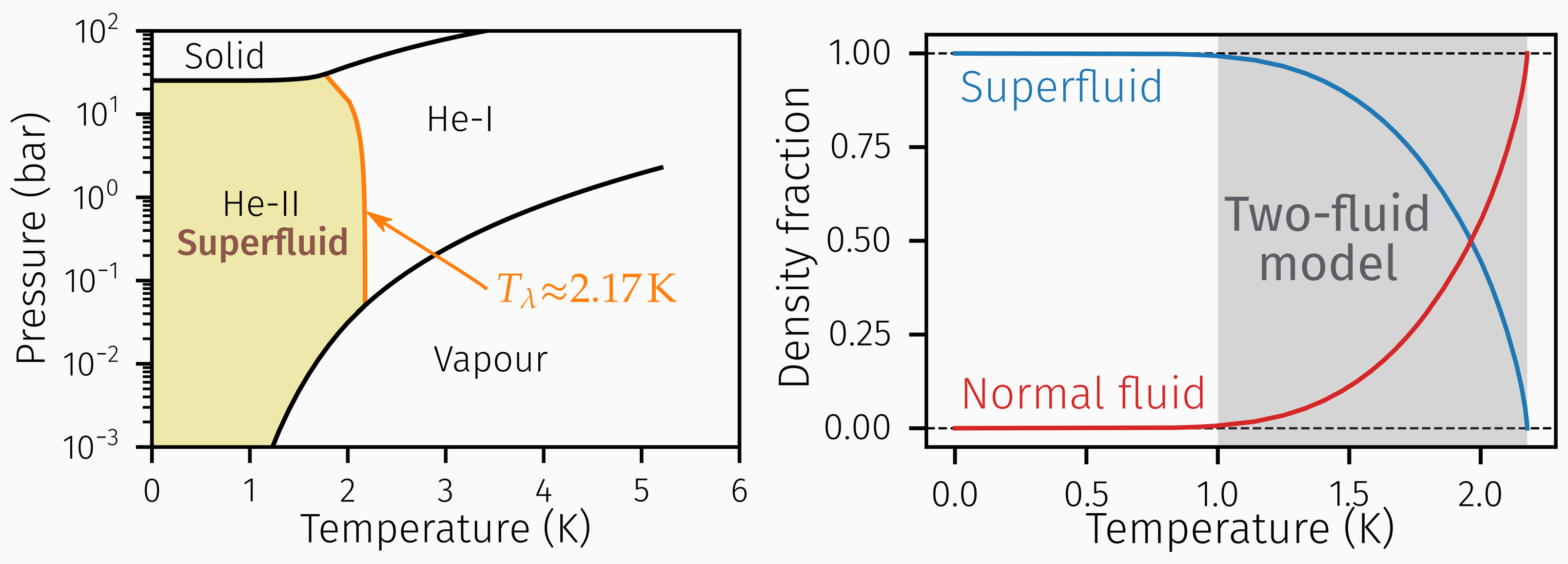

Figure 1. **Left:** Phase diagram of ^{4}He. **Right:** Relative densities of both components.

Counterflow phenomenon

- Produced by a **temperature gradient** in superfluid helium
- Normal and superfluid components flow with **mean relative velocity** $\boldsymbol{U}_{ns}$
- Very efficient **heat transport** by normal component

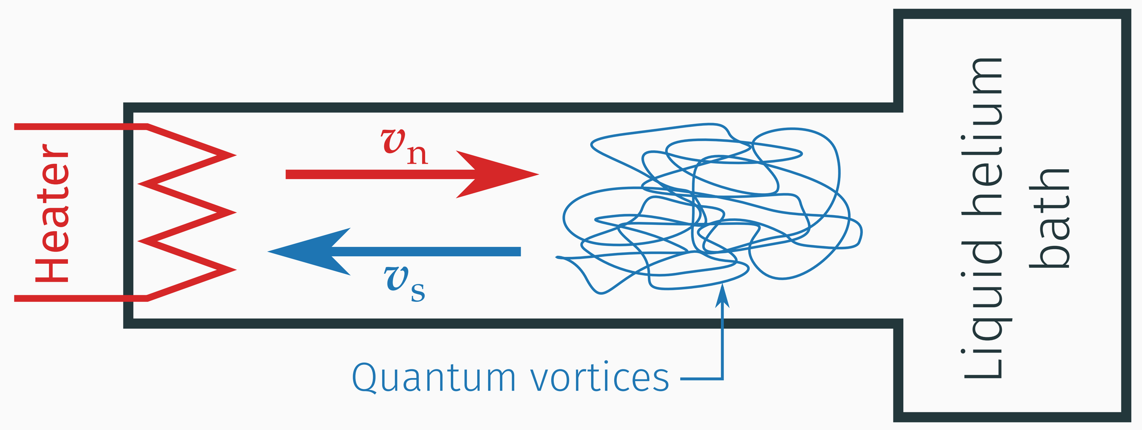

Figure 2. Counterflow in channel of superfluid helium.

Hall–Vinen–Bekarevich–Khalatnikov model

Two-fluid model at macroscopic scales:

$$\frac{\partial \boldsymbol{v}_n}{\partial t} + (\boldsymbol{U}_n + \boldsymbol{v}_n)\cdot\nabla\boldsymbol{v}_n = -\frac{\nabla p_n}{\rho_n} + \nu_n \nabla^2 \boldsymbol{v}_n - \frac{\boldsymbol{F}_{ns}}{\rho_n} + \boldsymbol{\varphi}_n$$

$$\frac{\partial \boldsymbol{v}_s}{\partial t} + (\boldsymbol{U}_s + \boldsymbol{v}_s)\cdot\nabla\boldsymbol{v}_s = -\frac{\nabla p_s}{\rho_s} + \nu_s \nabla^2 \boldsymbol{v}_s + \frac{\boldsymbol{F}_{ns}}{\rho_s} + \boldsymbol{\varphi}_s$$

$$\nabla\cdot\boldsymbol{v}_n = \nabla\cdot\boldsymbol{v}_s = 0$$

$\boldsymbol{U}_n, \boldsymbol{U}_s$ mean velocity of each component ($\Rightarrow \boldsymbol{U}_{ns} = \boldsymbol{U}_n - \boldsymbol{U}_s$)
$\boldsymbol{v}_n, \boldsymbol{v}_s$ fluctuating velocity of each component
$\nu_s(T)$ effective superfluid viscosity → microscopic dissipation
$\boldsymbol{\varphi}_n, \boldsymbol{\varphi}_s$ external forcing $\sim$ Normal$(0, \sigma_f)$ at wave numbers $|\boldsymbol{k}| \approx k_f$

$\boldsymbol{F}_{ns} = \alpha\rho_s\Omega_0(\boldsymbol{v}_n - \boldsymbol{v}_s)$ mutual friction force
$\Omega_0 \sim \sqrt{\langle|\omega_s|^2\rangle}$ mutual friction frequency
$\alpha(T)$ temperature-dependent mutual friction parameter

Bidimensionalisation under intense counterflow

At strong counterflow velocity $\boldsymbol{U}_{ns}$:
→ system becomes **quasi-two-dimensional**
→ reminiscent of rotating turbulence, MHD, thin-layer flows, ...

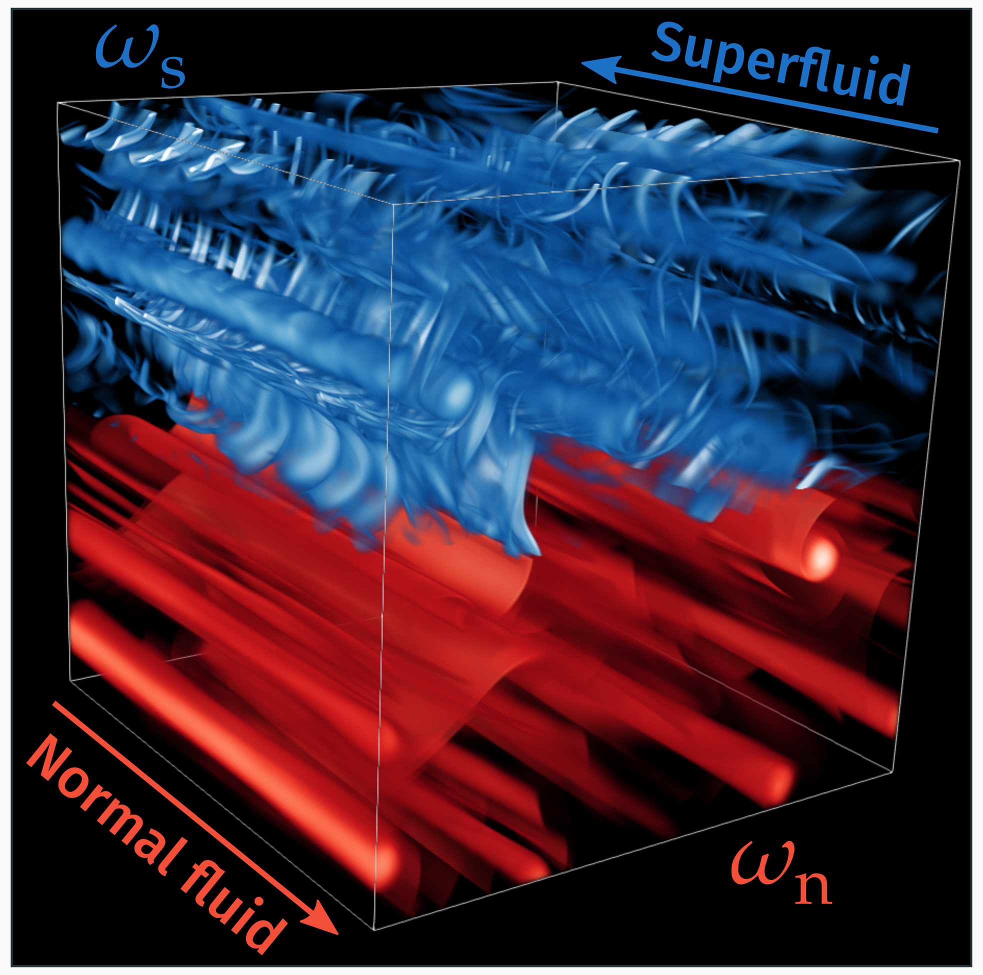

Figure 3. Statistically-steady 3D superfluid turbulence under strong counterflow at $T = 1.9$ K.

Emergence of a split energy cascade

Starting from initial state with zero fluctuations ($\boldsymbol{v}_n = \boldsymbol{v}_s = \boldsymbol{0}$):
→ **inverse energy cascade** for $k < k_f$, with $E(k) \sim k^{-5/3}$
→ **direct energy cascade** for $k > k_f$, with $E(k) \sim k^{-3}$
→ scalings compatible with phenomenology of **classical 2D turbulence**

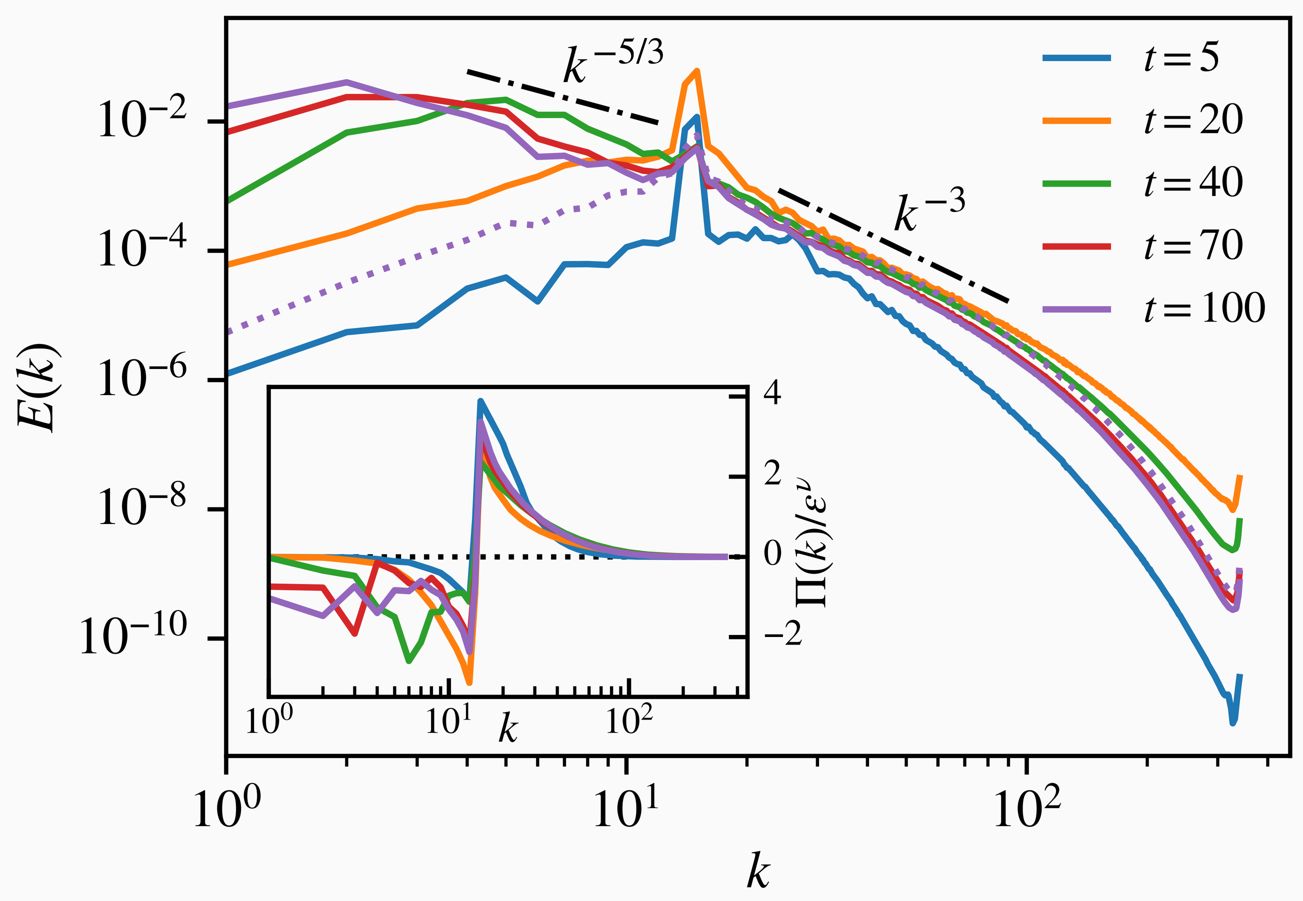

Figure 4. Temporal evolution of kinetic energy spectrum. Times are normalised by the forcing time scale $t_f = (k_f\sigma_f)^{-1/2}$. Inset: normalised energy flux. Simulation performed using $N^3 = 1024^3$ collocation points.

Abrupt transition towards split cascade scenario

Transition happens **abruptly** at critical counterflow velocity U^*_{ns}.

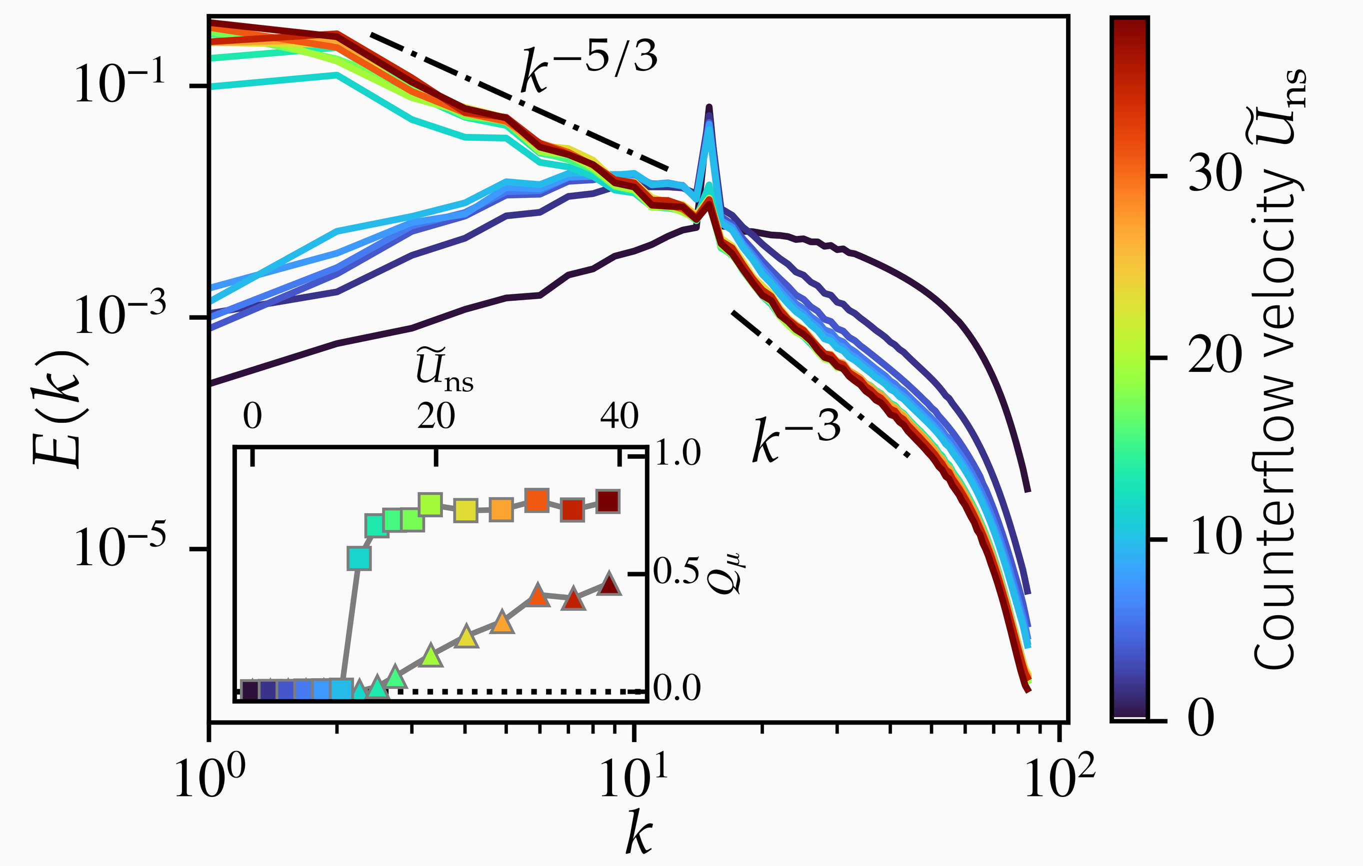

Figure 5. Kinetic energy spectrum for different counterflow velocities. Inset: relative large-scale dissipation using 2D (squares) and 3D (triangles) forcing schemes.

Here, simulations are performed with a **large-scale dissipation term** and with **hyperviscous** small-scale dissipation.

A **2D forcing scheme** is used to obtain a cleaner quasi-2D state at large U_{ns}.

The **critical counterflow velocity** U^*_{ns} can be expressed in terms of the **forcing** and the **mutual friction** parameters.

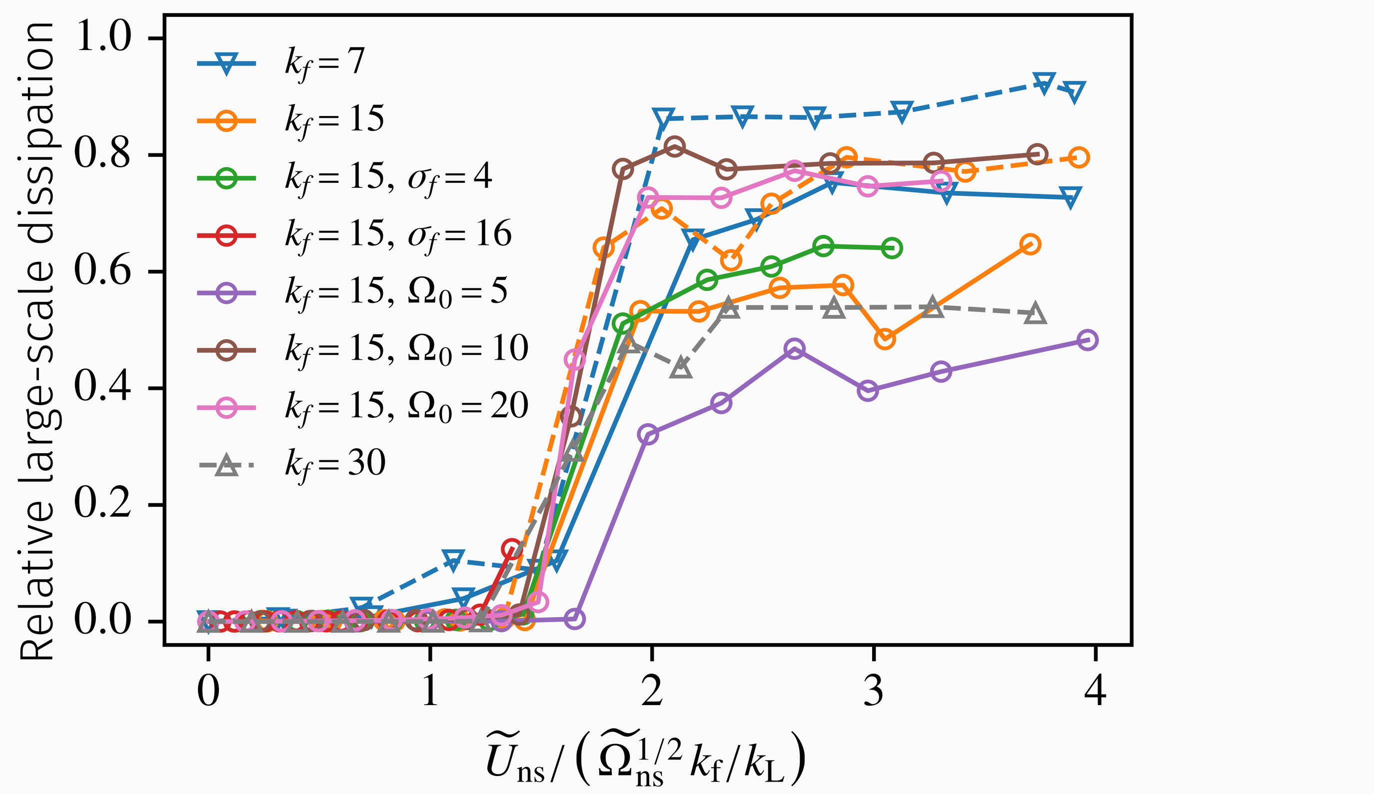

Figure 6. Relative large-scale dissipation Q_μ for different forcing and mutual friction parameters.

Perspectives

- Motivate **experimental study** of (quasi-)2D turbulence in superfluid helium.
- The **physical origin** of this transition is not currently understood.
- Characterisation of **temperature effects** and **hysteresis**.
- Application to other **two-fluid systems** (e.g. partially-ionised MHD).