Beyond the tip of the parametric instability tongue

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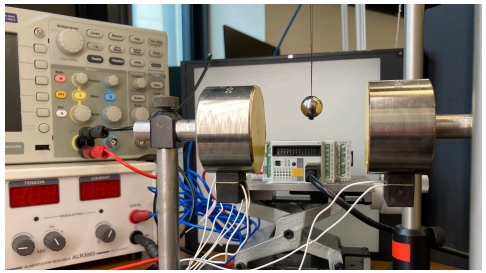
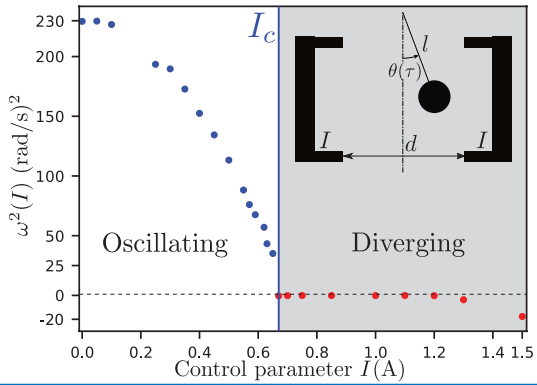
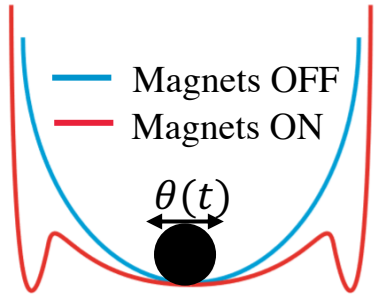
By periodically modulating the evolution law of a dynamical system one can enhance and trigger high order parametric instabilities. By modifying the period of modulation T one can obtain exciting new applications.

Electromagnetic pendulum



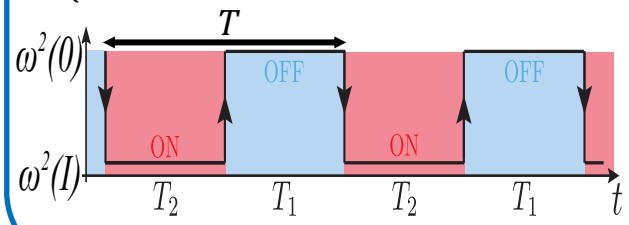




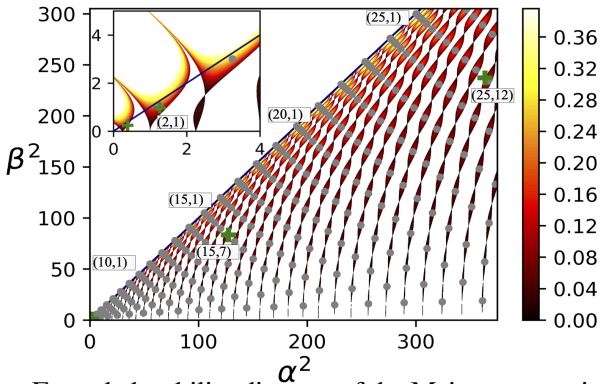
Extreme parametric instabilities

Linear two-state oscillator:

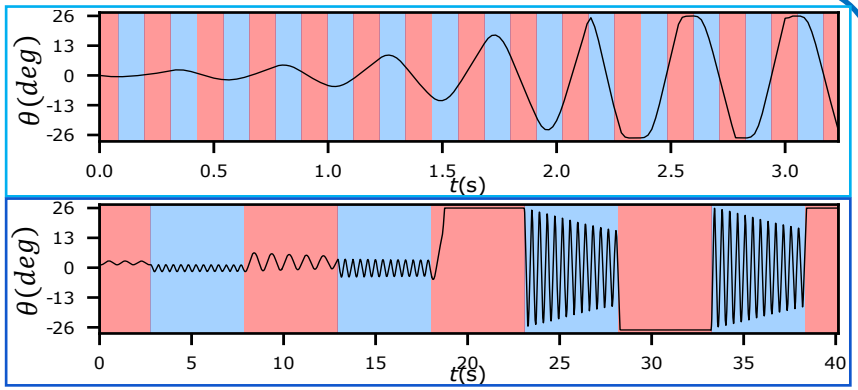
$$\begin{cases} \ddot{\theta}(t) + \omega^2(0)\theta(t) = 0 \ \text{during}\ T_1, \\ \ddot{\theta}(t) + \omega^2(I)\theta(t) = 0 \ \text{during}\ T_2, \end{cases}$$



Square wave modulation: $T_1 = T_2$



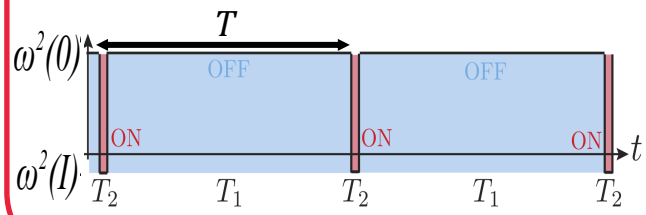
Extended stability diagram of the Meissner equation



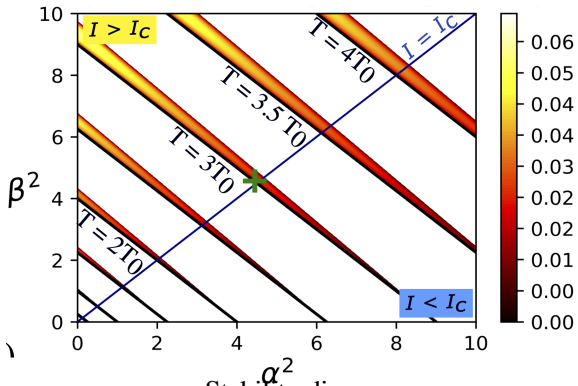
Experimental observation of the 2nd and 36th instability tongue

Triggering & sustaining a natural oscillation

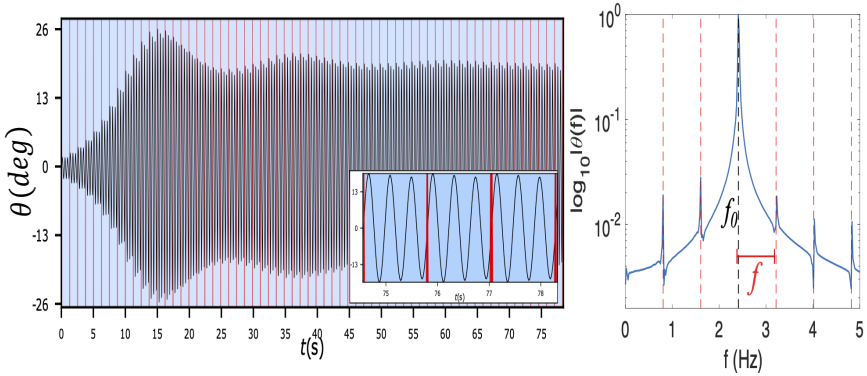
The "natural" oscillation of a system with period T_0 can be obtain as long as $T \approx 0.5kT_0,\ k \in \mathbb{N}^*$



"Impulse train" modulation: $T_1 \gg T_2$



Stability diagram



Experimental natural sinusoidal motion triggered and sustained every $T \approx 3T_0$

Reference: Grandi, A. A., Protière, S., & Lazarus, A. (2021). Enhancing and controlling parametric instabilities in mechanical systems. *Extreme Mechanics Letters*, *43*, 101195.