Confined vs. Unconfined: Triadic Resonant Instability in Cylindrical Geometry

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Key Points

- Cylindrical modes are defined by (ω, l, m, p);
- They always satisfy the IW dispersion relation;
- Sub-harmonics may be created, forming a triad;
- In unconfined geometry, we show that all of the four TRI conditions are exactly satisfied;
- In confined geometry, we show that there is an approximate TRI since the spatial resonance may not be satisfied due to the boundary conditions;
- When approximate, the quality of the resonance can be estimated thanks to an asymptotic expansion of the Bessel functions.

Introduction

Non-linear triads of internal waves have been studied in two-dimensional Cartesian geometry, where the Triadic Resonant Instability (TRI) description exhibits resonance conditions on the wave frequencies and wave numbers. In cylindrical geometry, internal waves are described as Kelvin modes, that can be defined through their vertical velocity field

$$v_z(r, \theta, z, t) = v_z^0 J_p(lr) e^{i(\omega t - mz - p\theta)},$$

with ω the wave frequency and l, m, and p the radial, vertical, and azimuthal wave numbers. The formation of triads is investigated in the unconfined and in the confined configurations.

Experimental Setup

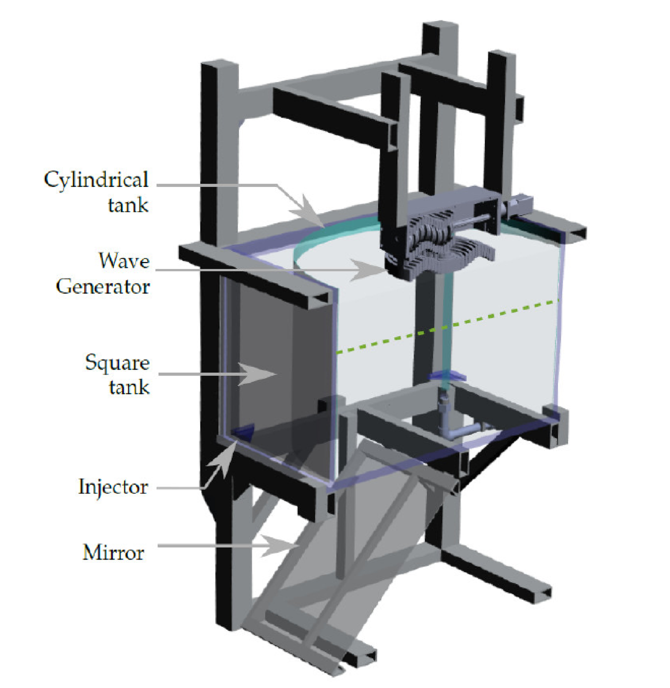

Figure 1: Experimental setup used to produce the wave fields [3]. A confining cylinder (same radius as generator) can be added.

Unconfined Wave Fields

	Physical domain	Wave numbers	Resonances
Temporal	$t \in \mathbb{R}$	$\omega \in \mathbb{R}$	$\omega_0 = \pm\omega_1 \pm \omega_2$
Radial	$r \in \mathbb{R}^+$	$l \in \mathbb{R}$	$l_0 = \pm l_1 \pm l_2$
Azimuthal	$\theta \in [0;\, 2\pi]$	$p \in \mathbb{N}$	$p_0 = \pm p_1 \pm p_2$
Vertical	$z \in \mathbb{R}$	$m \in \mathbb{R}$	$m_0 = \pm m_1 \pm m_2$

Table 1: TRI in unconfined domain, with exact resonance conditions.

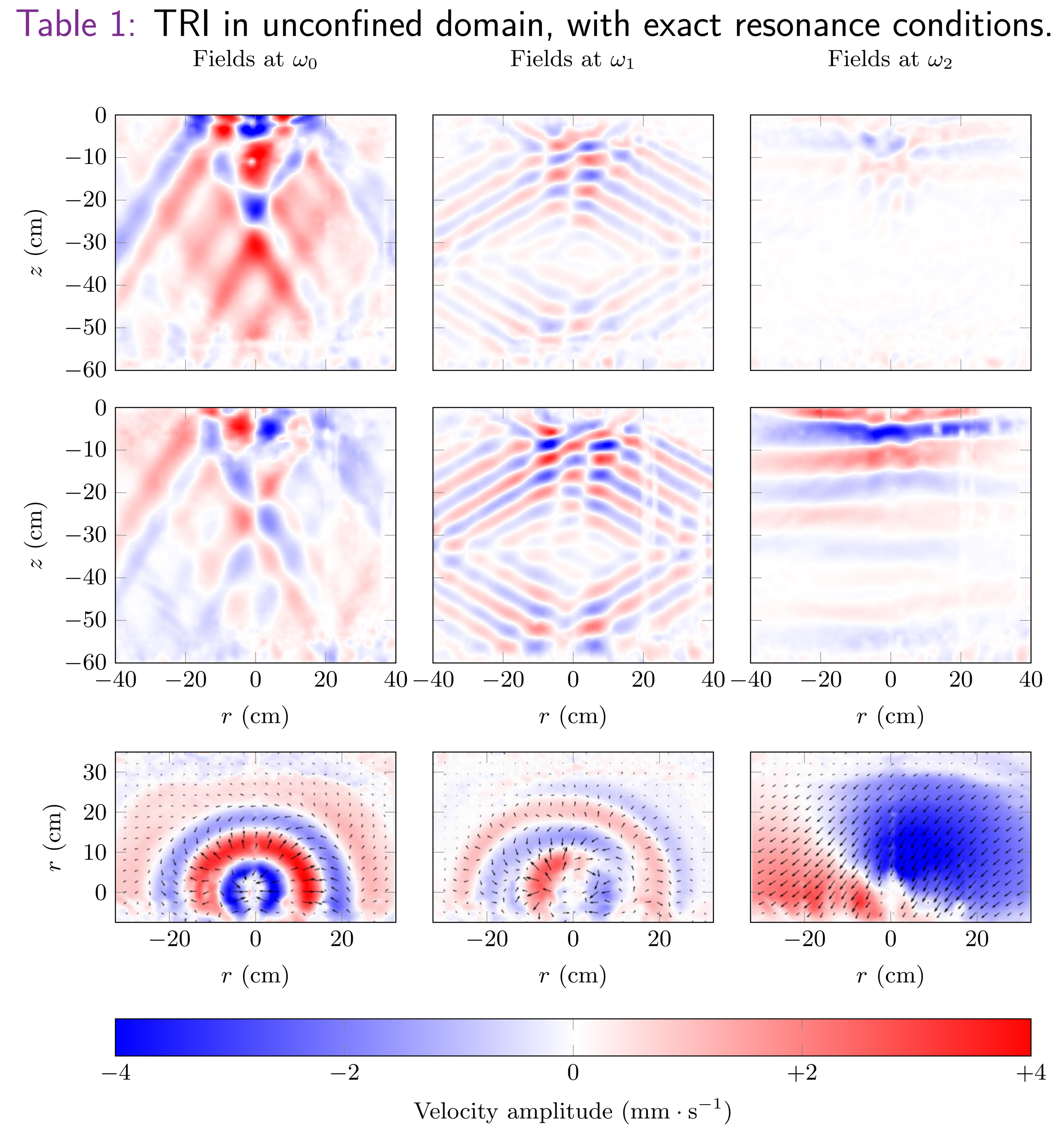

Figure 2: Unconfined velocity fields of the triad, filtered at the three wave frequencies. Experiment at $f = 0.29\ \mathrm{rad \cdot s^{-1}}$, $N = 0.97\ \mathrm{rad \cdot s^{-1}}$.

We measure:

- $\omega_0 = 0.80\ \mathrm{s^{-1}}$, $\omega_1 = 0.50\ \mathrm{s^{-1}}$, and $\omega_2 = 0.30\ \mathrm{s^{-1}} \to \omega_0 = \omega_1 + \omega_2$.
- $m_0 = 16\ \mathrm{m^{-1}}$, $m_1 = 63\ \mathrm{m^{-1}}$, and $m_2 = 49\ \mathrm{m^{-1}} \to m_0 = m_1 - m_2$.
- $p_0 = 0$, $p_1 = +1$, and $p_2 = -1 \to p_0 = p_1 + p_2$.
- $l_0 = 42\ \mathrm{m^{-1}}$, $l_1 = 31\ \mathrm{m^{-1}}$, and $l_2 = 11\ \mathrm{m^{-1}} \to l_0 = l_1 + l_2$.

Confined Wave Fields

	Physical domain	Wave numbers	Resonances
Temporal	$t \in \mathbb{R}$	$\omega \in \mathbb{R}$	$\omega_0 = \pm\omega_1 \pm \omega_2$
Radial	$r \in [0;\, R]$	$l \times R$ a Bessel zero	$l_0 \simeq \pm l_1 \pm l_2$
Azimuthal	$\theta \in [0;\, 2\pi]$	$p \in \mathbb{N}$	$p_0 = \pm p_1 \pm p_2$
Vertical	$z \in [0;\, H]$	$m = n\pi/(2H), n \in \mathbb{N}$	$m_0 = \pm m_1 \pm m_2$

Table 2: TRI in confined domain, with approximate resonance conditions on l.

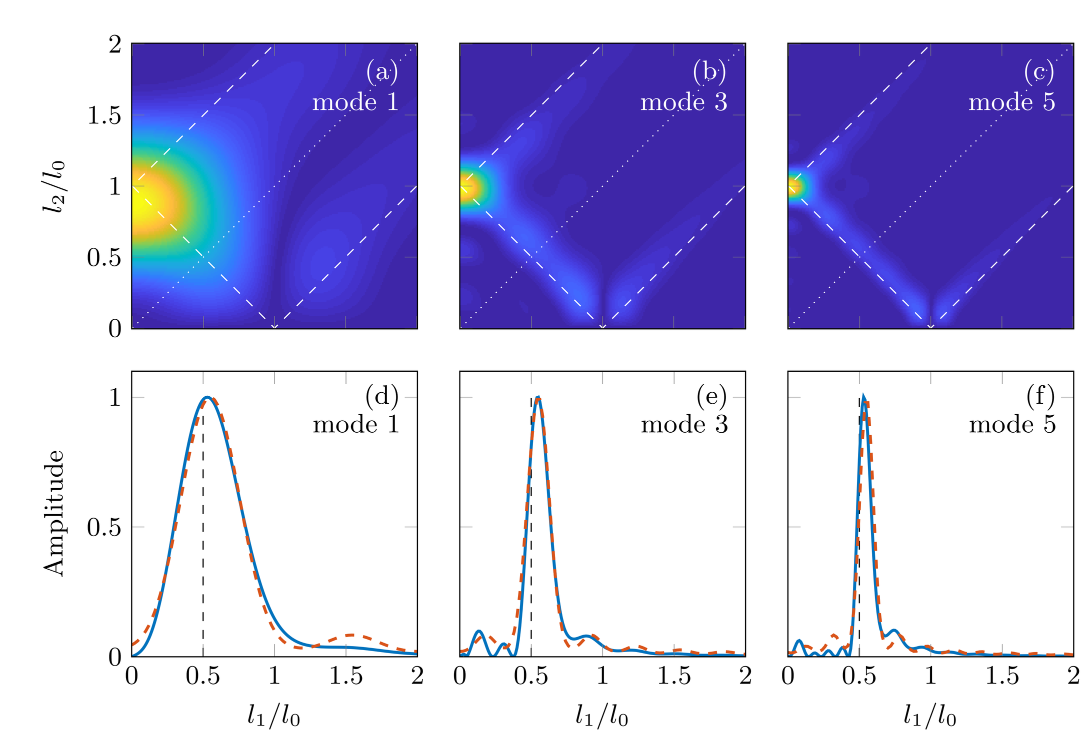

Figure 3: Graphic characterisation of the radial resonance in confined domains.

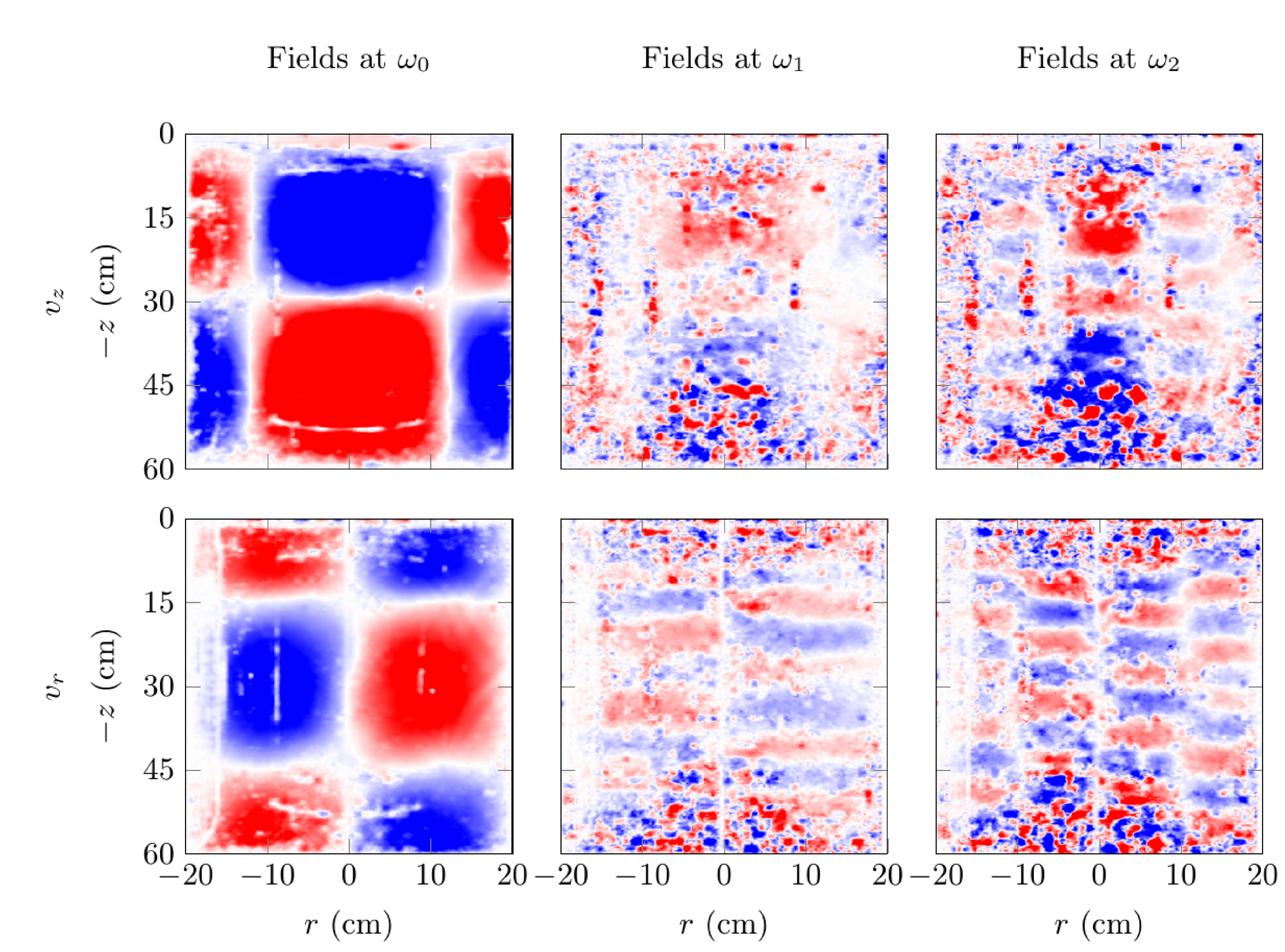

Figure 4: Confined velocity fields of the triad, filtered at the three wave frequencies. Experiment at $f = 0\ \mathrm{rad \cdot s^{-1}}$, $N = 0.81\ \mathrm{rad \cdot s^{-1}}$, with $\omega_0 = 0.74\ \mathrm{rad \cdot s^{-1}}$, $\omega_1 = 0.30\ \mathrm{rad \cdot s^{-1}}$, and $\omega_2 = 0.44\ \mathrm{rad \cdot s^{-1}}$; we have a frequency resonance $\omega_0 = \omega_1 + \omega_2$.

References

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