Rare event-triggered transitions in aerodynamic bifurcation

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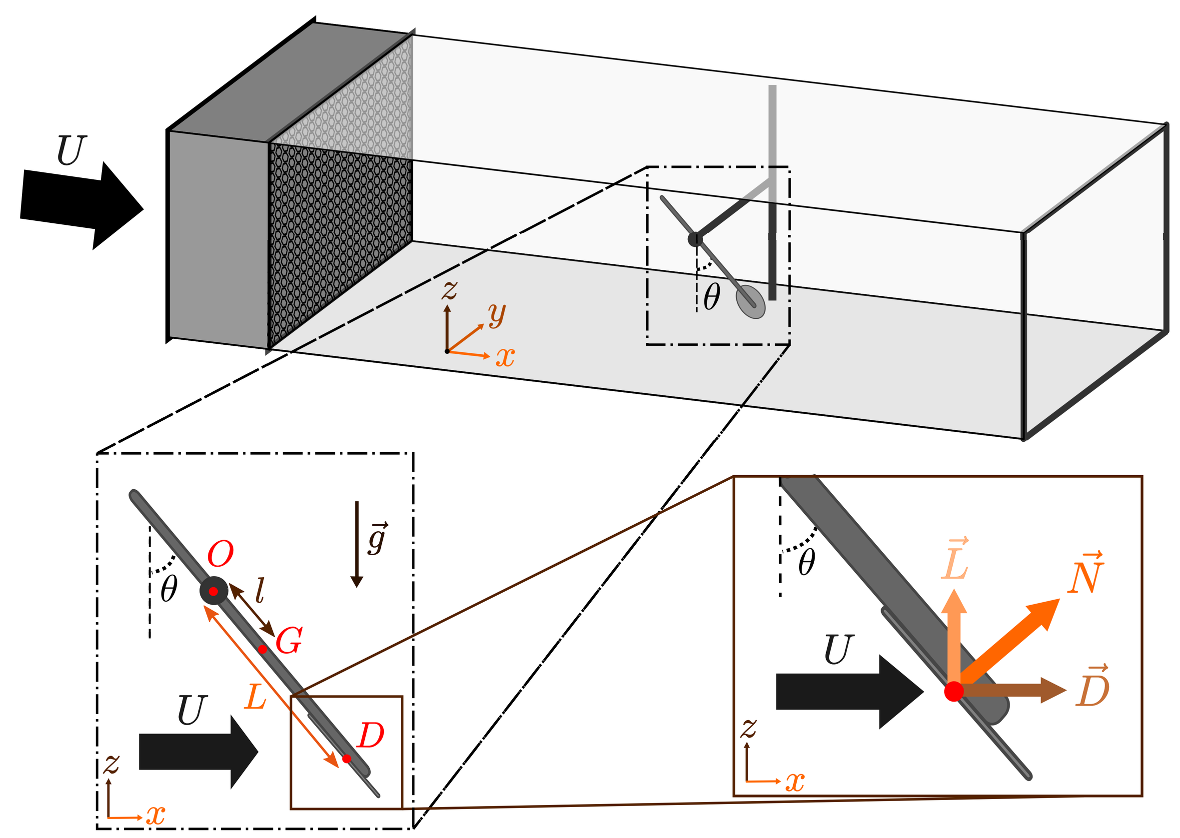
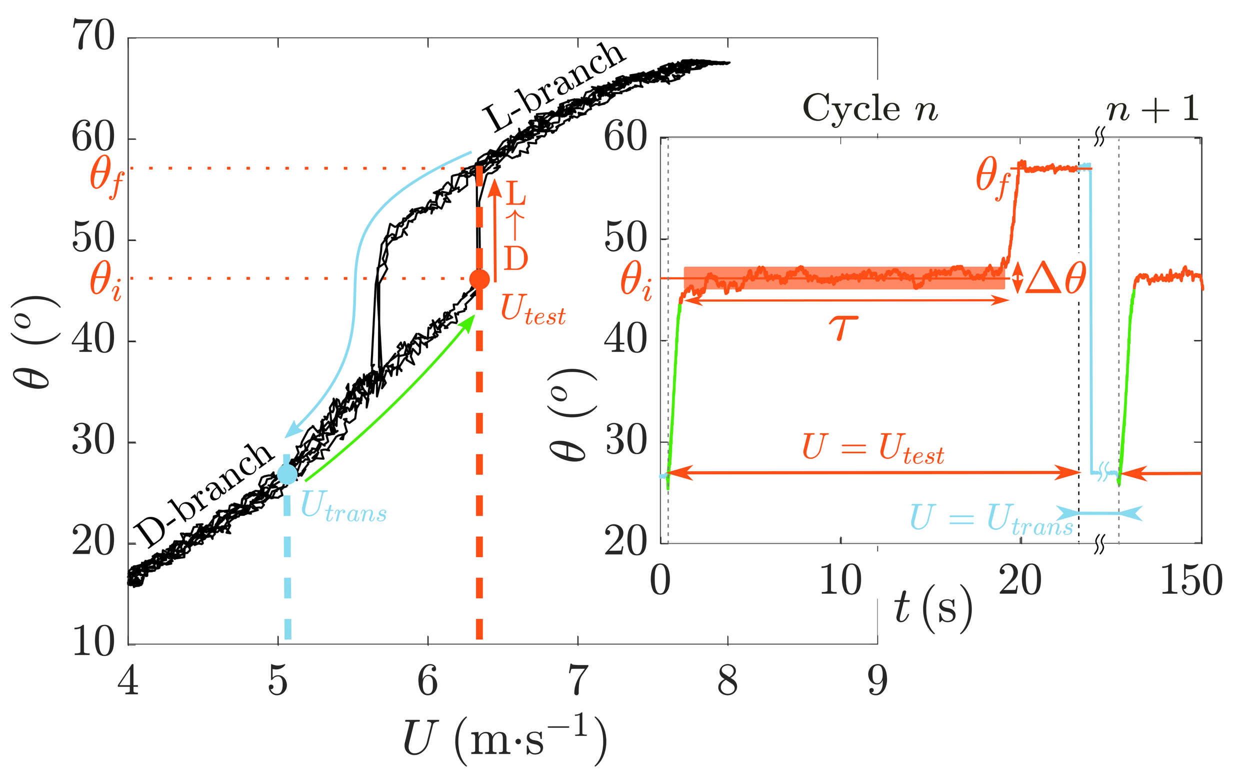
1·Introduction

The dynamics of a simple pendulum in a wind tunnel flow, of mean velocity U, exhibit a complex hysteritic behavior [*Obligado et al.* 2013]. This hysteretic behavior comes from a predominance of Lift ($\vec{L}$) or Drag ($\vec{D}$) in the Normal force $\vec{N}$, which balances the Weight $m\vec{g}$.



$$J\ddot{\theta} = -mg\overline{OG}\sin(\theta) + \frac{1}{2}\rho S C_N(\theta) U^2 \overline{OD}$$



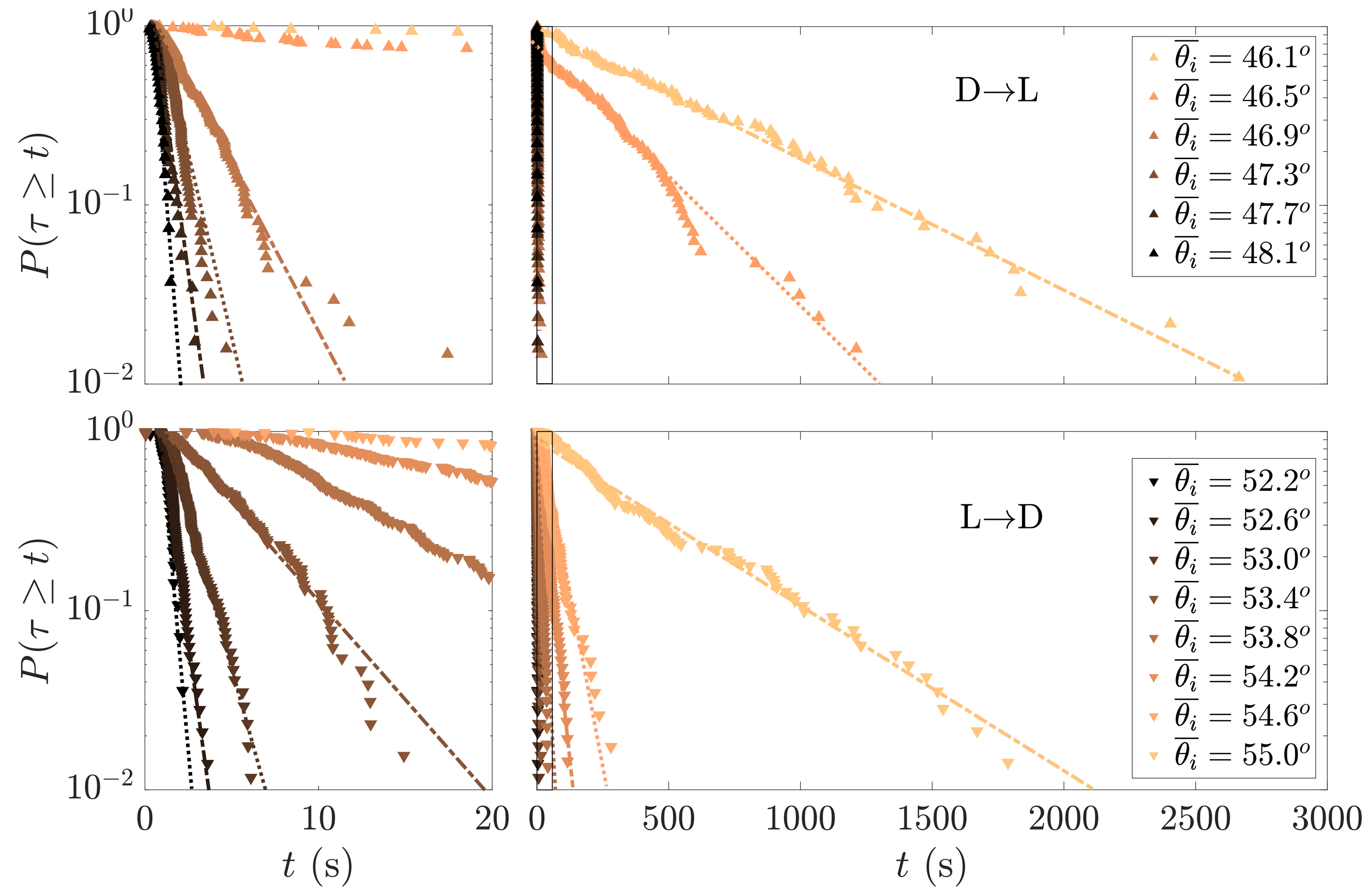
In the hysteretic cycle, spontaneous transitions are observed.
We investigate them in details by a systematic protocol:

- Resetting the flow for 2 minutes at U_{trans},
- Increasing the flow velocity U to its test value,
- Waiting for the transition to happen at U_{test},
- Going back to U_{trans} and starting again.

Such protocol enables the study of upward (D→L) and downward (L→D) transitions, depending on U_{trans} and U_{test}.

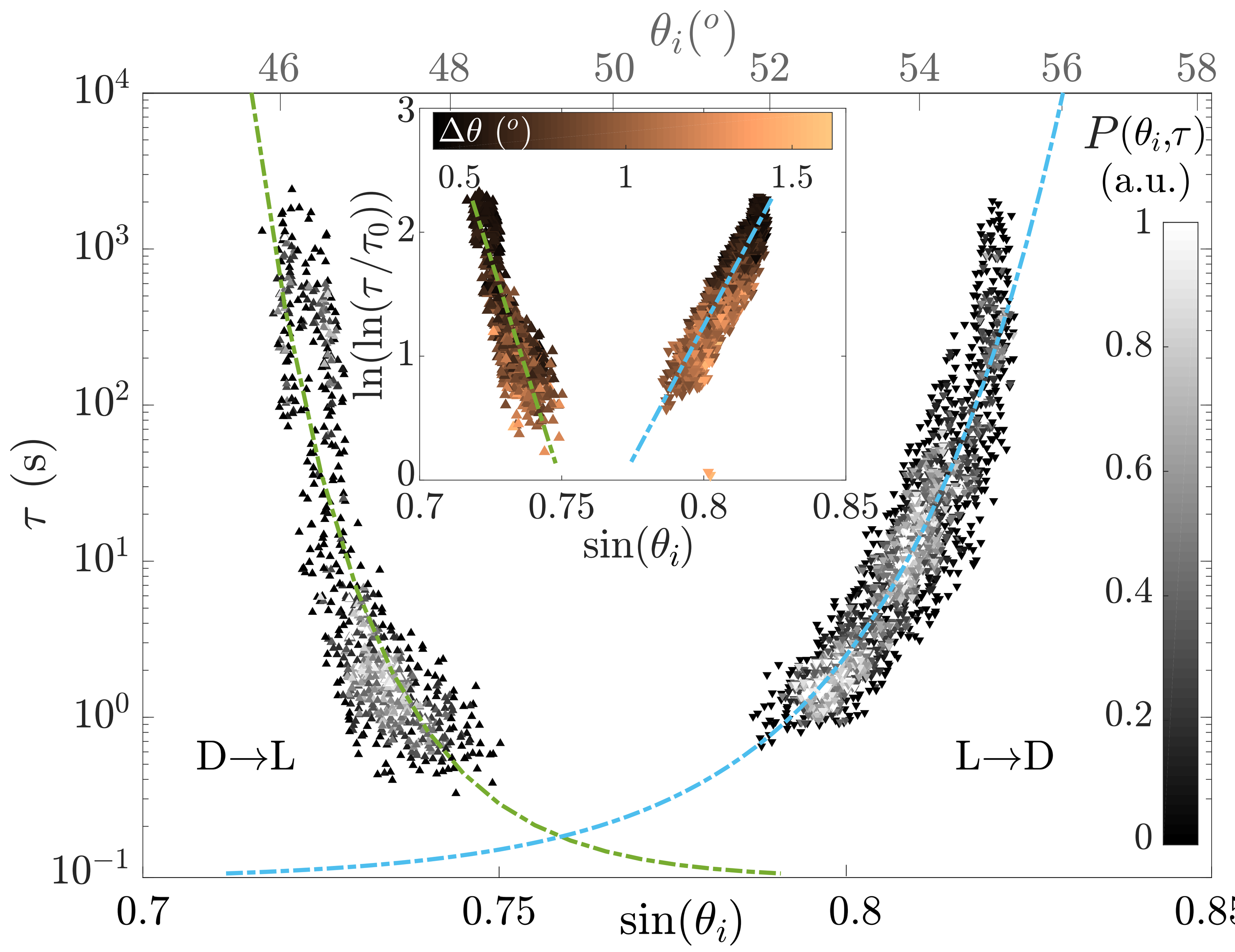
2·Statistics

When reproducing a large number of transitions, we obtain exponential distributions of waiting times τ for a given initial angle $\overline{\theta_i}$.



Assembling all statistics without restraining the value of θ_i presents strong dependencies of τ as a function of $\sin(\theta_i)$, which can be fitted with double-exponential:

$$\tau = \tau_0 \exp\left[\exp\left(\frac{\sin(\theta_i) - \sin(\theta_0)}{\eta}\right)\right].$$



Such double-exponential functions are also observed in the transition to turbulence, and can be explained by Gumbel distributions of the maxima of turbulent kinetic energy [Goldenfeld et al. 2010].

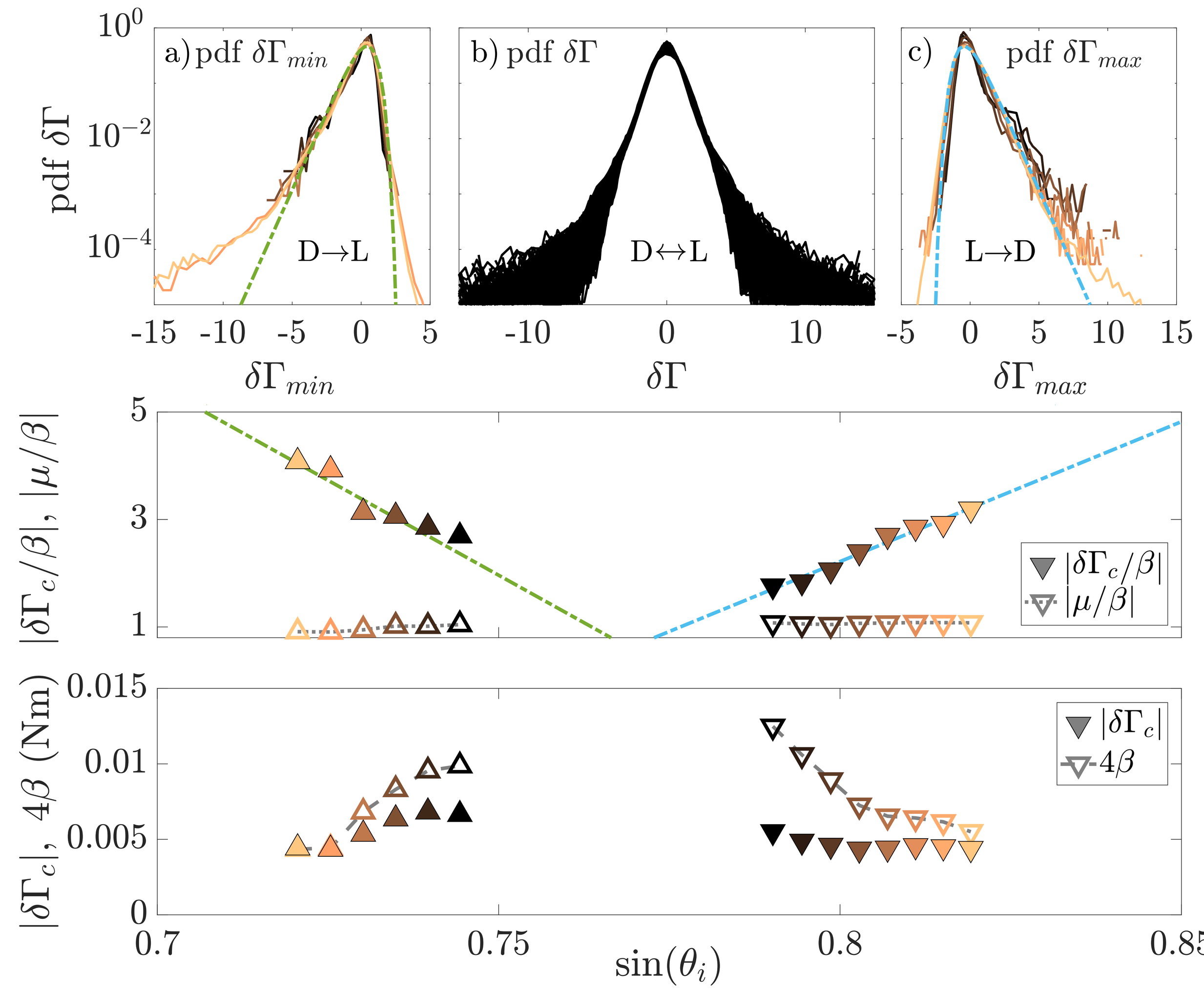
3·Model

From the theory of the transition to turbulence, we link the transitions to rare events, extreme fluctuations in the aerodynamic torque:

$$\delta\Gamma = \Gamma_{aero}(t) - \tfrac{1}{2}\rho S C_N(\theta) U^2 \overline{OD}$$

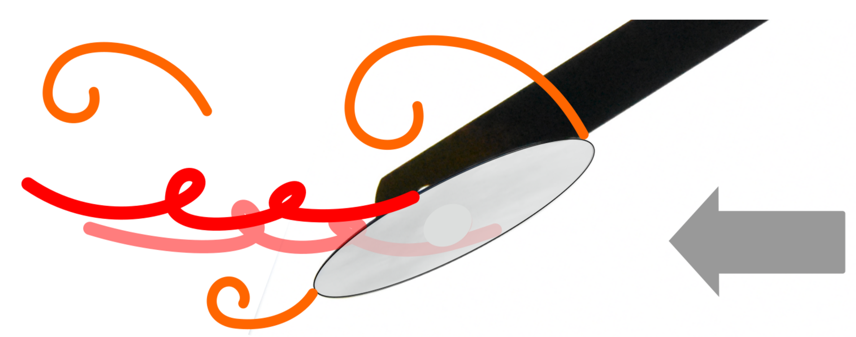
The pendulum experiences a transition if the fluctuations of torque are below a threshold $\delta\Gamma_c$. This threshold can be estimated from the parameters (μ,β) of the Gumbel distributions of $\delta\Gamma$ and its average waiting time τ_c.

$$\frac{\tau_0}{\tau_c(\overline{\theta_i})} = \exp\left[-\exp\left(-\frac{\delta\Gamma_c(\overline{\theta_i}) - \mu(\overline{\theta_i})}{\beta(\overline{\theta_i})}\right)\right]$$



4·Conclusion

The rare events in the aerodynamic torque triggering the transitions are expected to take place in the wake of the pendulum. In particular, τ_0 is associated to a Strouhal number $St = 0.07$ reported to be linked with transverse vortex shedding.



References

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