# How laboratory rivers transport sediment

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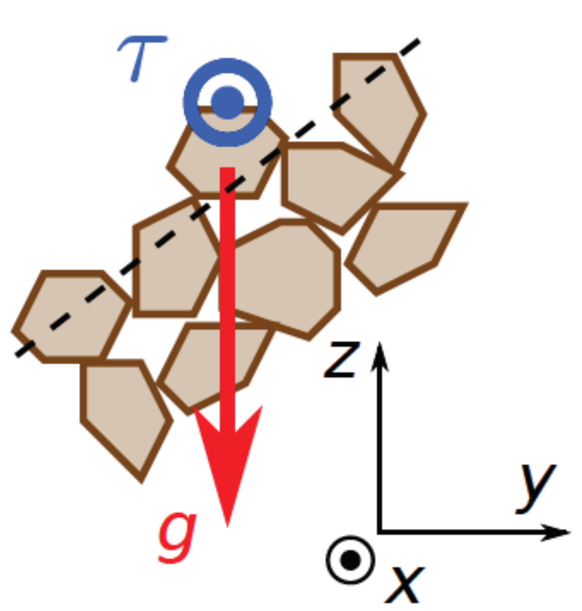
## 1. Introduction

Sediment transport by rivers shapes much of the landscape on Earth. Yet, how rivers adjust their shape to enable sediment transport is not understood.

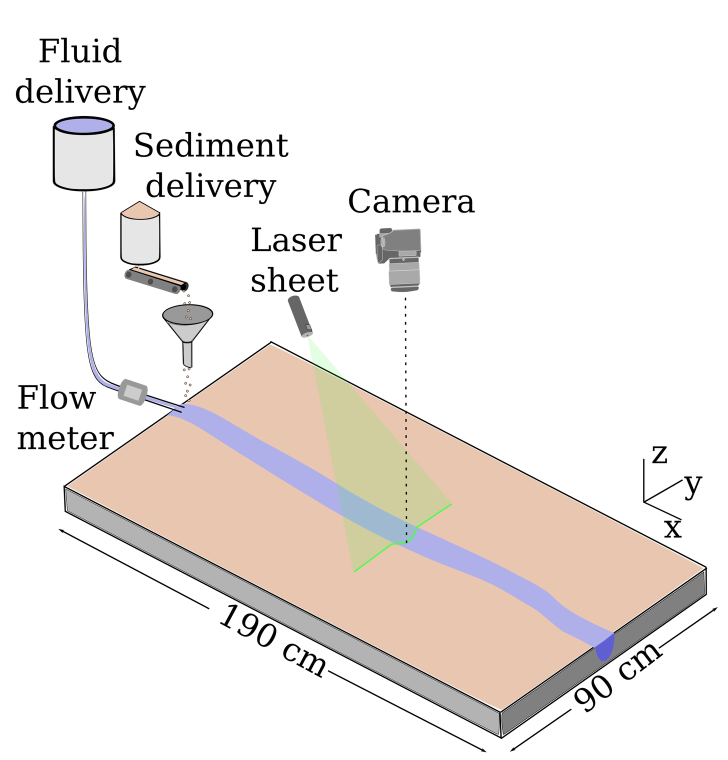
## 2. Rivers exist near the threshold of sediment transport




*(Seizilles et al. 2013)*

Bed of sediment rivers typically exists close to the threshold of sediment motion - below threshold it would not possible to move the sediment grains and build the bed, while significantly above the threshold, the bed would quickly erode away. Any sediment transported by rivers is driven by small force deviations from this threshold. Therein lies the main challenge for understanding sediment transport in rivers - to find the shape of a river carrying a certain amount of sediment, we have find the water stress, which sensitively depends on the river shape itself, with high accuracy.

## 3. Experiment




We considered the experiments of Abramian et al. 2020. In these experiments, straight laminar rivers formed in a bed of plastic sediment. Fluid (a mixture of water and glycerol) and sediment were fed into the river at prescribed rates. River cross-section was monitored by a laser sheath, and sediment flux profile (sediment discharge per unit river width) could be estimated by grain counting in movies filmed by an overhead camera.

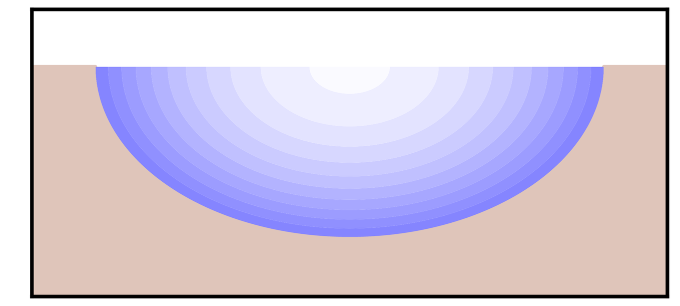
River shape, sediment flux profile, and downstream slope all robustly adjusted to the discharges of fluid and sediment.

## 4. Mechanisms for river formation

### 1. Stokes flow

The laminar flow in the channel obeys the Stokes law:

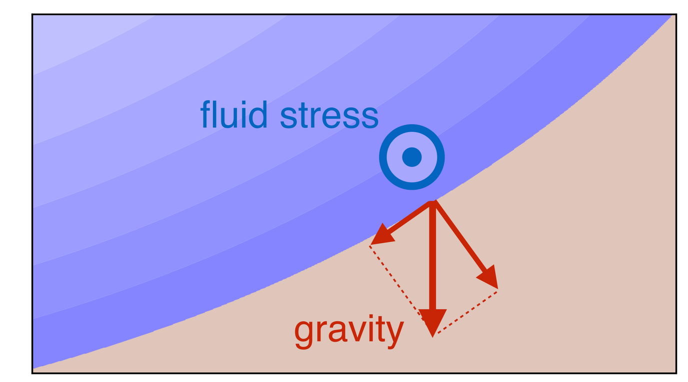
$$\nu \Delta u = -gS$$



### 2. Sediment flux

Grains on the sediment bed feel the force of gravity and the fluid shear stress. Sediment flux, $q_s$, depends on the ratio, $\mu$, of forces tangential to the bed that act to move the grains to normal forces that act to keep them in place. When the force ratio is below a threshold, $\mu_t$, there is no sediment flux, while above, sediment flux is proportional to the distance to threshold:

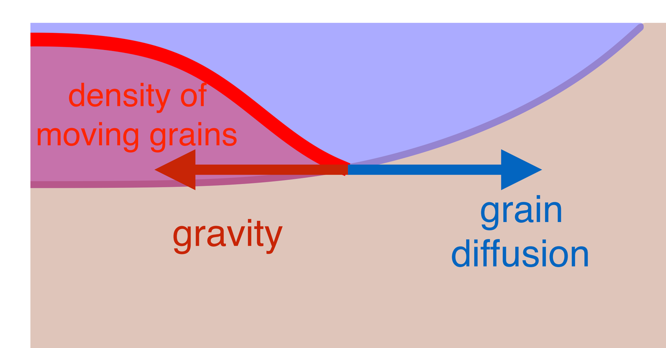
$$q_s = q_\mu(\mu - \mu_t)$$




### 3. Sediment diffusion

Due to random interactions with the river bed, sediment traveling downstream also diffuses across stream from parts where sediment transport is intense to quieter parts. Analogous to a gas in hydrostatic balance in a gravitational field, balance between grain diffusion and gravity leads to a Boltzmann-like distribution of moving grains as a function of the flow depth, $D$:

$$q_s = q_\mu e^{(D-\beta)/\lambda}$$




## 5. Our model for the channel shape

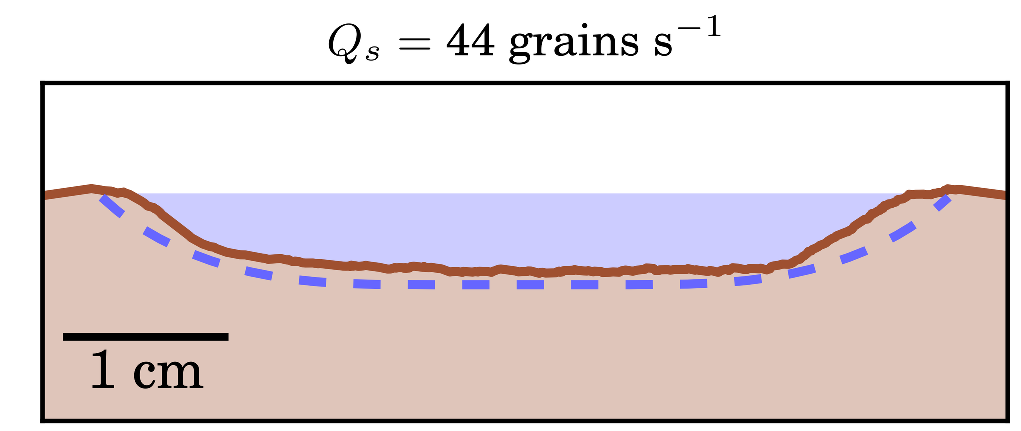
Based on the above, we develop a model for the shape of the river cross-section. First, Stokes flow (Eq. 1) relates the river shape to the fluid stress on the bed. Then, stress on the bed determines the sediment flux through Eq. 2. Finally, sediment flux is related to the bed shape through the Boltzmann equation (Eq. 3).

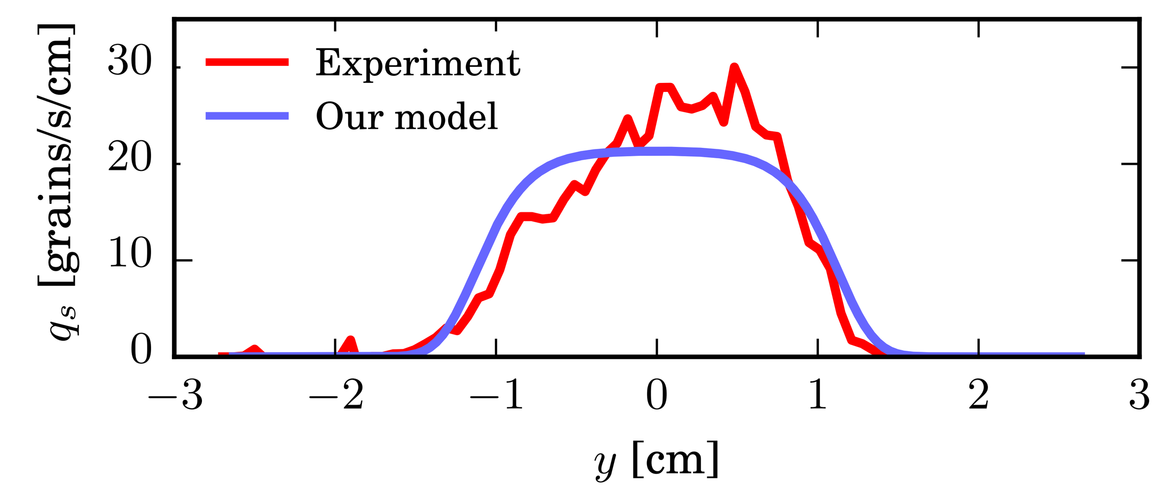
We simplify the problem by assuming the river is much wider than it is deep, which leads to a second order differential equation for the river shape:

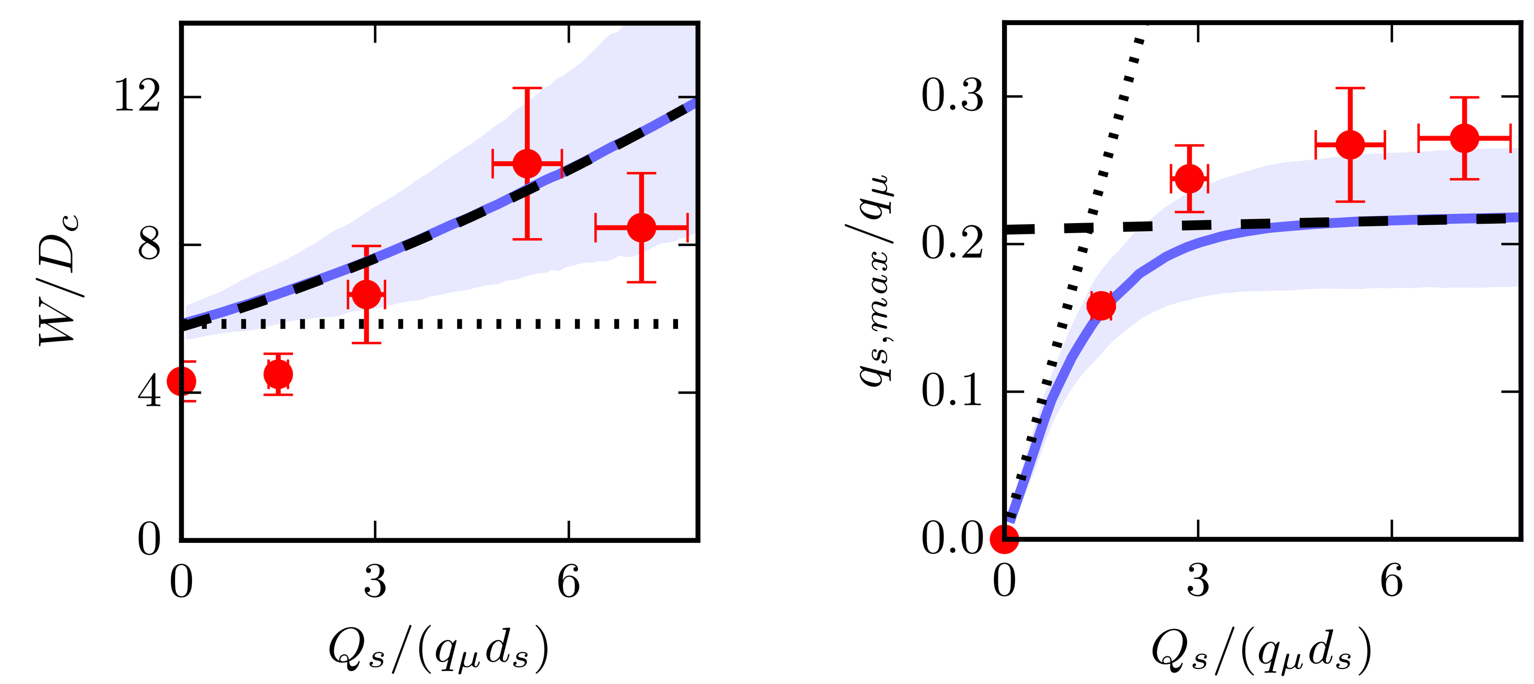
$$\sqrt{\frac{S^2}{L_s^2}\left(D + \frac{1}{3}(D^3)''\right)^2 + D'^2} - \mu_t = e^{\frac{1}{\lambda}(D-\beta)}$$

$D$: shallow-water; $\frac{1}{3}(D^3)''$: cross-stream momentum diffusion; $D'^2$: gravity; $e^{\frac{1}{\lambda}(D-\beta)}$: Sediment diffusion

## 6. Our model matches the experiment with no tuning



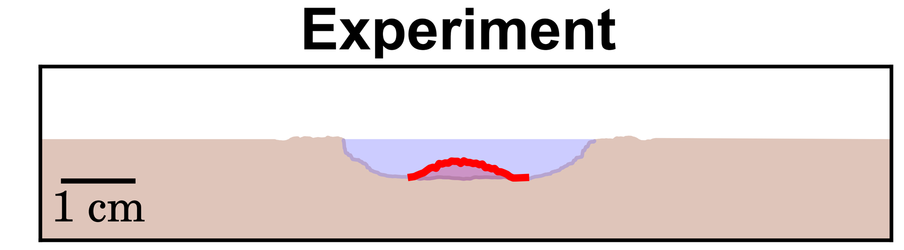




All of the parameters in our model are well-constrained in the experiments of Abramian et al. 2020. Rivers in our model reproduce the experiments well in all metrics with no tuning.

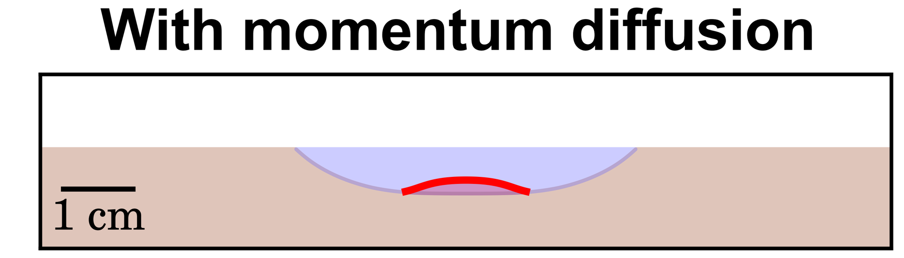
Our model predicts a maximum sediment flux of about $0.2q_\mu$ consistent with observations of natural rivers.

## 7. Cross-stream momentum diffusion is essential!

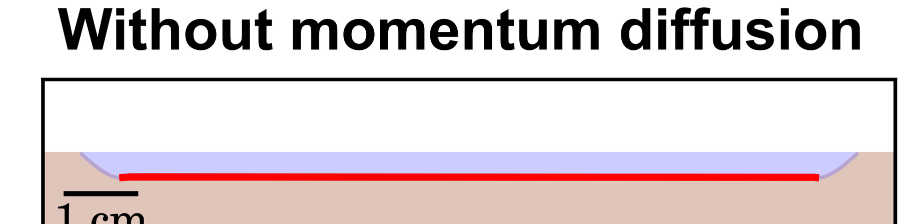
### Experiment



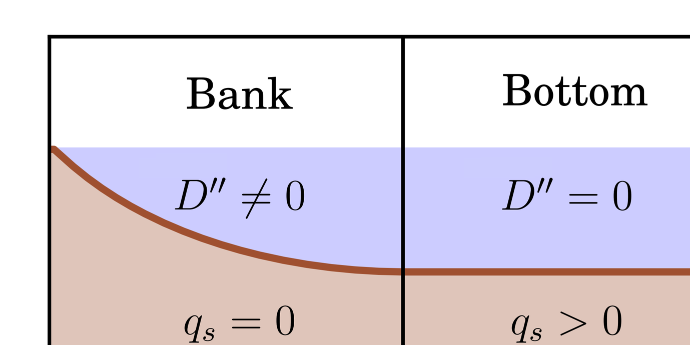

### With momentum diffusion




### Without momentum diffusion




Excluding momentum diffusion from our model, drastically changes the shape of the river. To explain the observations, we need to keep the momentum diffusion. Sediment transport by momentum diffusion is enabled by the sudden transition from curved banks to a flat bottom.

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