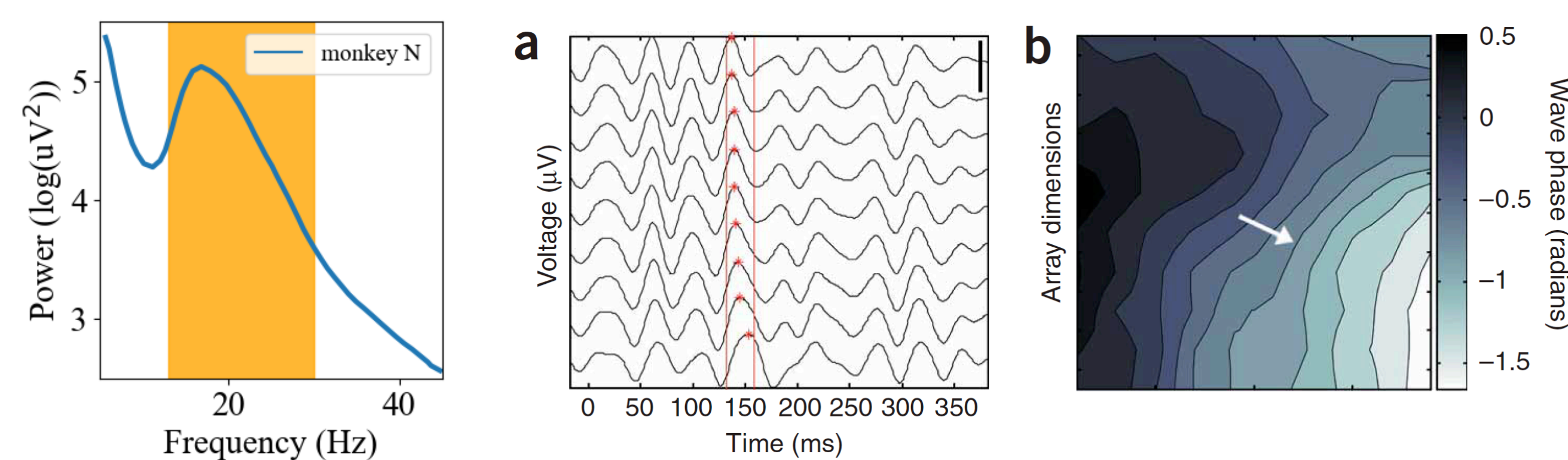
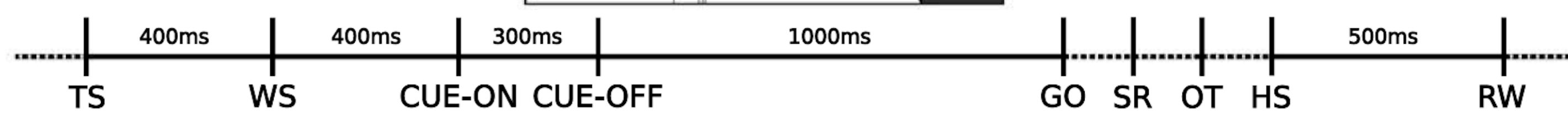
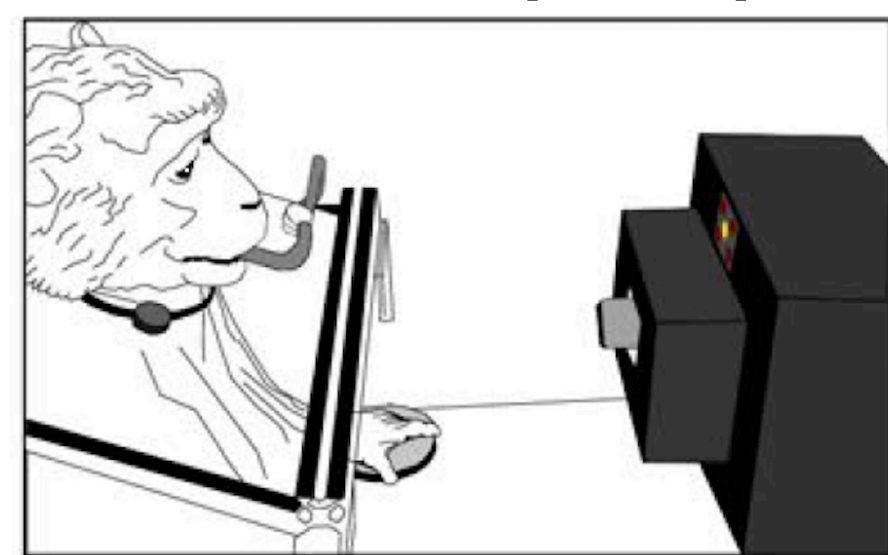


Abstract

Modeling the nonlinear dynamics of neural networks can help to make sense of phenomena observed in neural recordings. Here, we focus on beta frequency ($\sim 20\text{Hz}$) oscillations that are observed in motor cortex during movement preparation [1]. In several experiments, local field potentials (LFPs) recorded with multi-electrode arrays have been observed to display transient oscillations with non-zero phase shifts between electrodes. They organize into a variety of traveling waves (planar, radial, rotating,...) [2, 3, 4]. Beta oscillations have been successfully modeled [5] as arising from reciprocal interactions between randomly connected excitatory (E) pyramidal cells and local inhibitory interneurons (I). The synchronization properties of distant modules produced by distant-dependent excitatory coupling have also been studied [5, 6]. Here, we use a rate model (mean-field) description of the local neural activity that has been shown to provide an accurate population-level description of more detailed network simulations based on coupled spiking neurons [6]. Using distance-dependent interactions and delays matching those reported from experiments, we study this model in 2D to investigate possible origins of transient bursts of beta oscillations and the observed spatial waves. Stochastic local entries that vary on a long-time scale (200ms) are introduced to mimic inputs to the motor cortex from other neural areas. We compare our simulation results to electrophysiological datasets recorded in motor cortex of macaque monkey during an instructed delayed reach-to-grasp task [4]. We find that our model closely agrees with the recordings. It reproduces the observed power spectrum of the local field potential, produces a variety of traveling waves of speed and types similar to those seen in experiments. Our results suggest that both time-varying external entries and intrinsic network architecture shape the LFP dynamics of motor cortex.

Propagating waves in motor cortex

Beta oscillations are observed in motor cortex during the preparatory phase of movement. For monkeys trained to do an instructed delayed reach-to-grasp task, local field potentials (LFPs) exhibit non-zero phase shifts on separate electrodes of a multi-electrode array and are organized into waves [2, 3, 4].



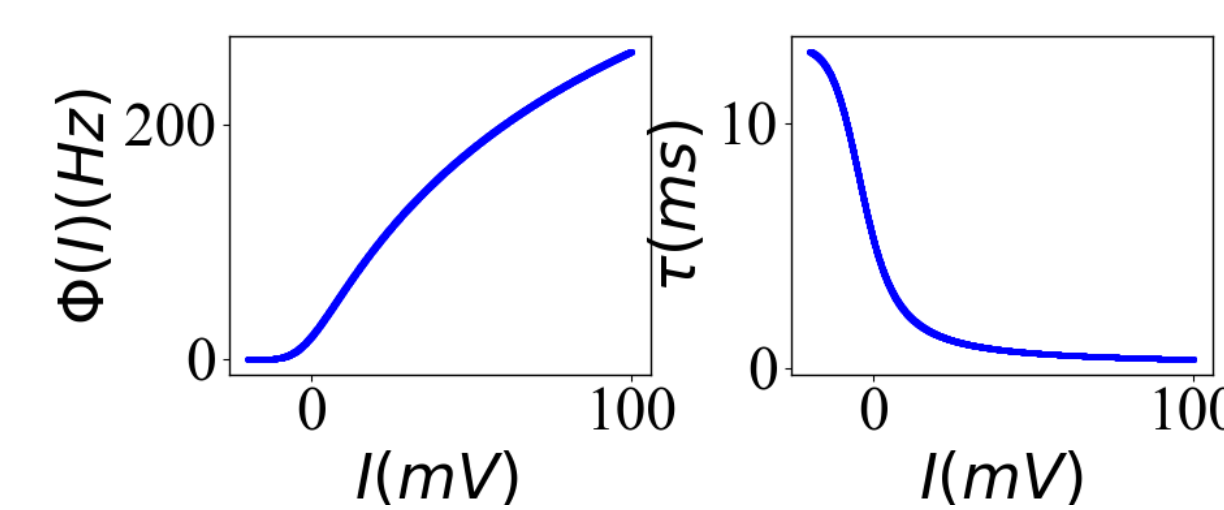
Figures reproduced from Rubino et al [2], Denker et al[4]

Rate-model description of an E-I module

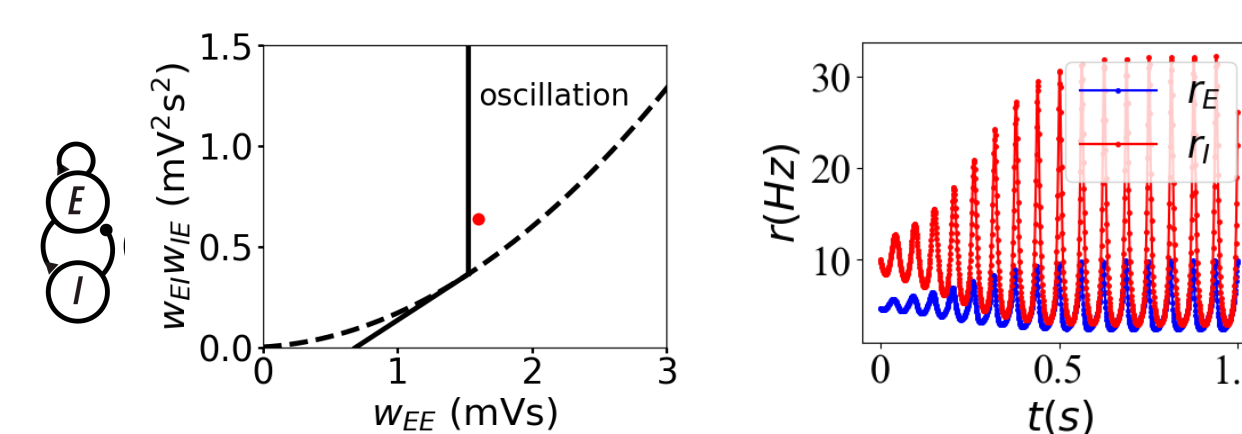
Beta oscillations are modeled as arising from the local interaction between an excitatory (E) and an inhibitory (I) neuronal population. Their activity is described using a rate model formalism [6]:

$$\begin{aligned} \tau_E(I_E) \frac{dI_E}{dt} &= -I_E + I_E^{ext} + w_{EE}r_E - w_{EI}r_I, \\ \tau_I(I_I) \frac{dI_I}{dt} &= -I_I + I_I^{ext} + w_{IE}r_E \\ r_E &= \Phi_E(I_E), r_I = \Phi_I(I_I) \end{aligned}$$

I_E and I_I are the mean inputs in each population. The adaptive time scales, τ_E and τ_I , and the neuron spiking rates, r_E and r_I , are obtained from the inputs by the following curves :



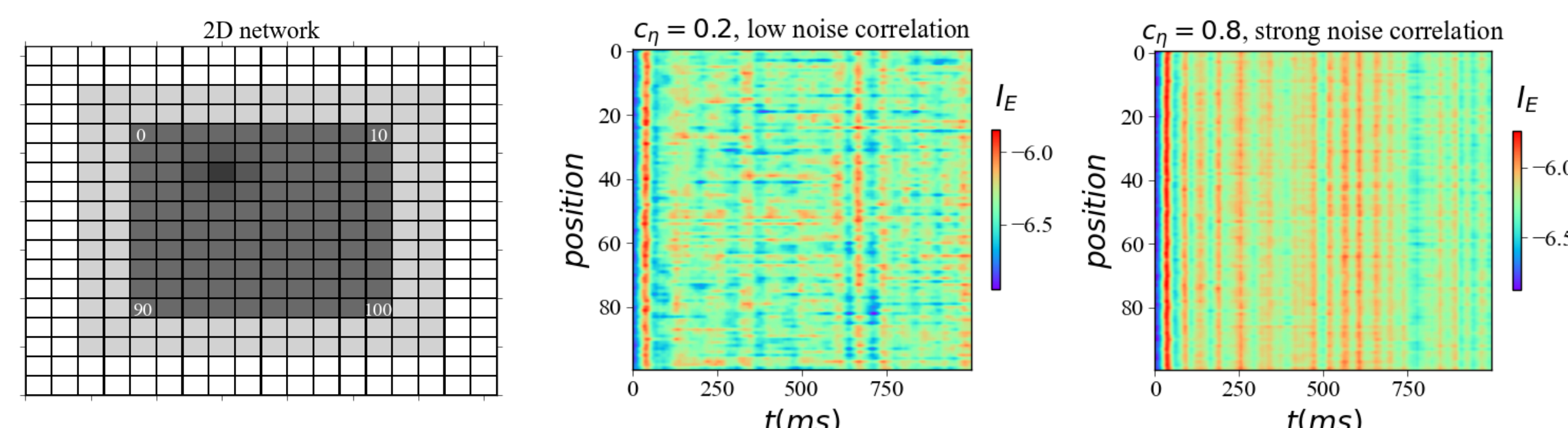
Phase diagram and example of oscillations of the rates (blue, r_E ; red, r_I ; parameters : red dot in the phase diagram):



Network model

In order to compare with the experimental results, we consider a 2d spatial network with a delayed interaction, and a Gaussian spatial profile, as well as **both local and global fluctuating inputs** :

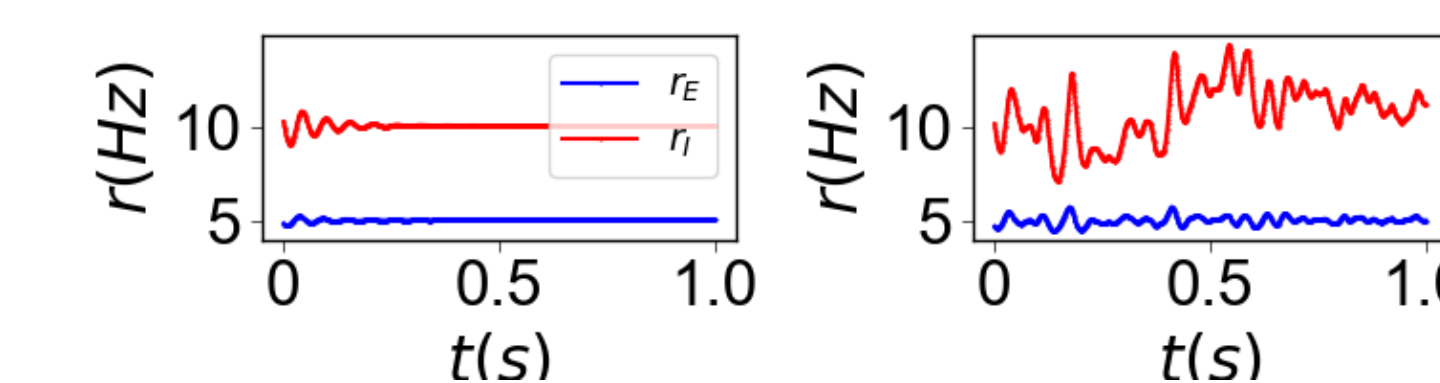
$$\begin{aligned} \tau_E(I_E)(n,t) \frac{\partial I_E(n,t)}{\partial t} &= -I_E(n,t) + I_E^{ext}(n,t) + w_{EE} \left[\sum_{m=1, \dots, L \times L} C(|m-n|) r_E(m, t-LD) \right] - w_{EI} r_I(n,t) \\ \tau_I(I_I)(n,t) \frac{\partial I_I(n,t)}{\partial t} &= -I_I(n,t) + I_I^{ext}(n,t) + w_{IE} \left[\sum_{m=1, \dots, L \times L} C(|m-n|) r_E(m, t-LD) \right] \\ I_{E,I}^{ext}(n) &= I_{E,I}^{ext,0} + \sigma_{E,I}^{ext} (\sqrt{c_\eta} \eta_{all} + \sqrt{1-c_\eta} \eta_m), \quad C(l) = \frac{1}{Z} \exp[-(l/\lambda)^2], \quad Z = \sum_l \exp[-(l/\lambda)^2] \end{aligned}$$



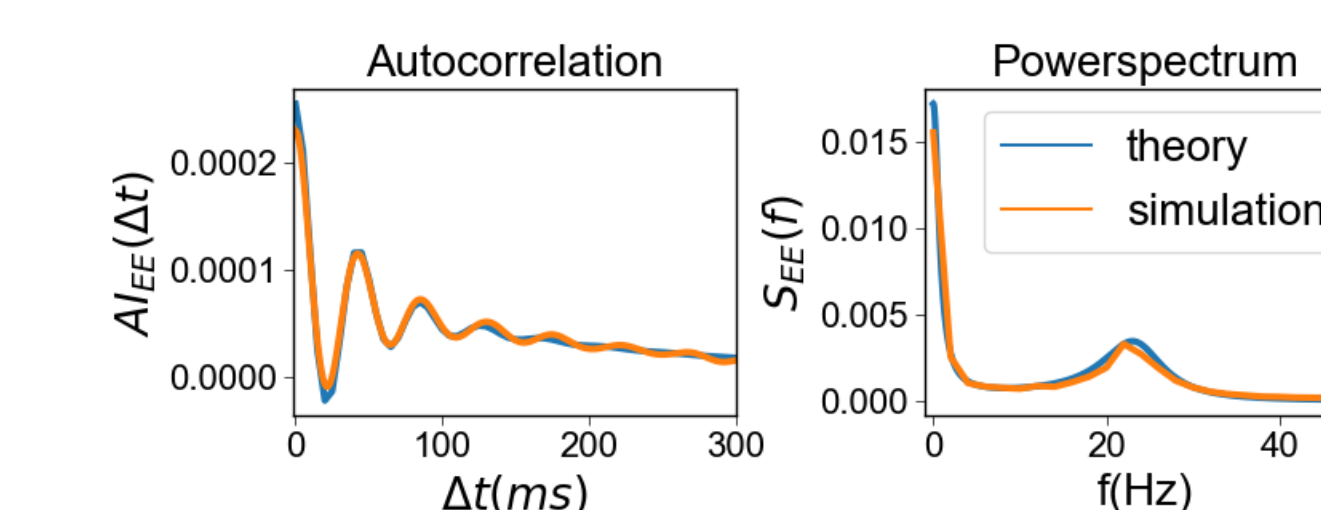
In experiments, recordings show a lot of power at very low frequencies. In order to model something similar, we consider external inputs that fluctuate on a long time scale described by η , where η is an O-U noise with a long (~ 200 ms) time constant τ_L .

$$\begin{aligned} I_E^{ext} &= I_E^{ext,0} + \sigma_E^{ext} \eta, \quad I_I^{ext} = I_I^{ext,0} + \sigma_I^{ext} \eta \\ \tau_L \frac{d\eta}{dt} &= -\eta + \xi, \quad \langle \xi(t) \xi(t') \rangle = \delta(t-t') \end{aligned}$$

Oscillations about the stable fixed point induced by the fluctuating inputs :

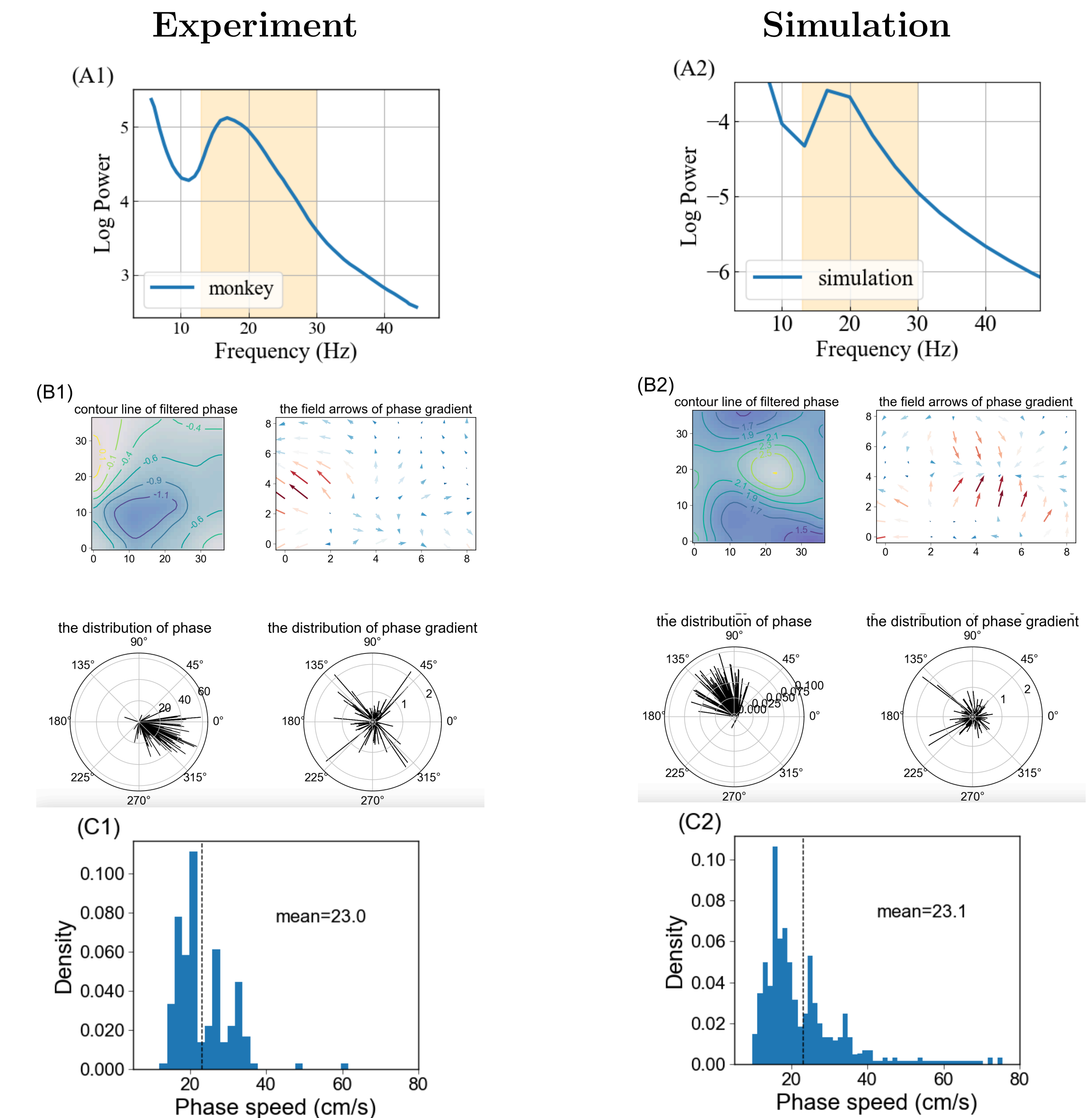


The power spectrum and the autocorrelation of the single module computed from linear response theory agree with the simulation results :



Comparison between simulation and experimental results

We observed similar power spectrum of the local field potential, a variety of traveling waves of speed and types to those seen in experiments.



Conclusions

- A simple model of interacting E-I modules with fluctuating inputs accounts for the experimental data
- It predicts that (thalamic?) inputs to the motor cortex should be both local and global.

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