Lagrangian compressible dynamics in a self-similar incompressible jet

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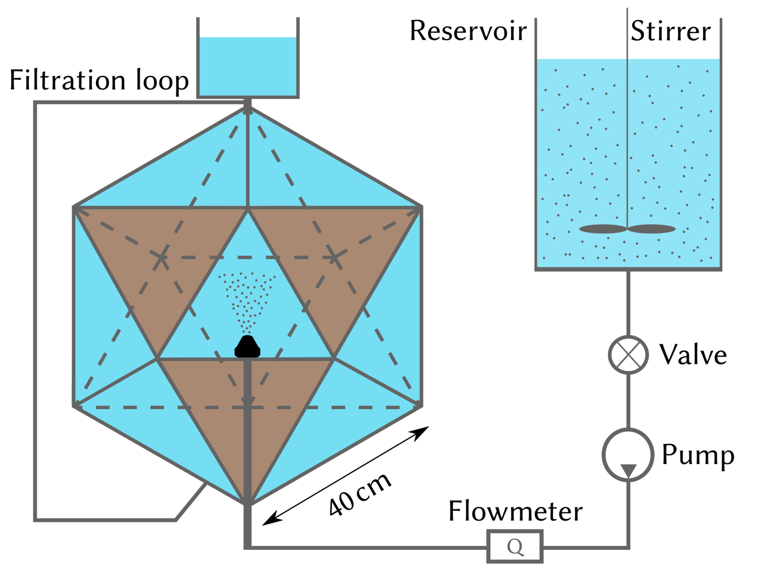
A turbulent water jet seeded with tracers

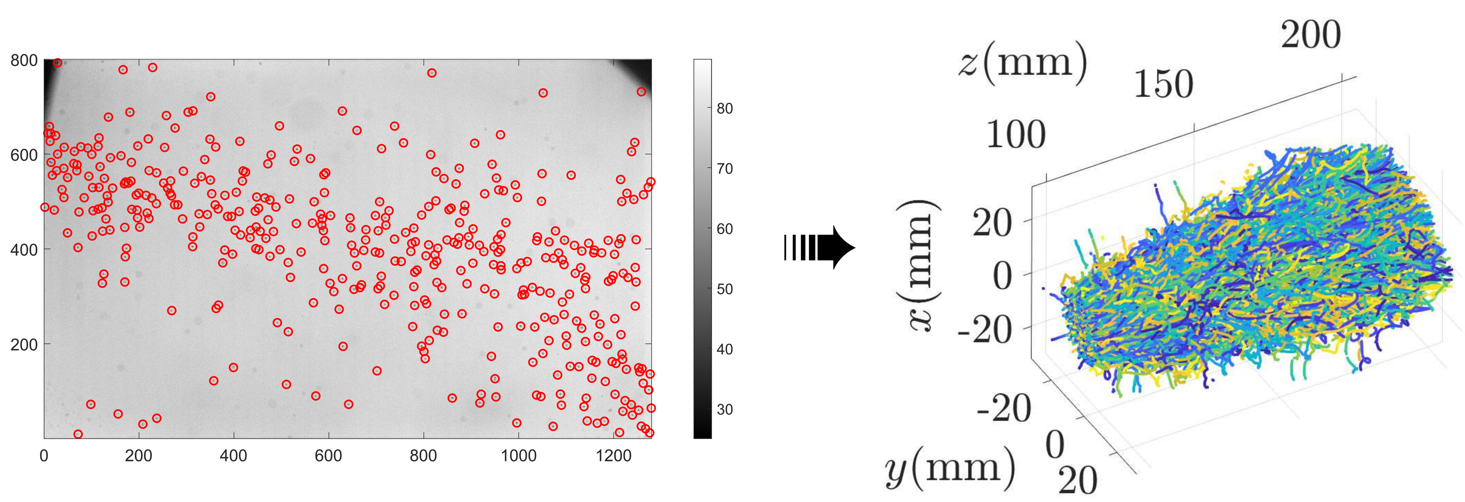


- A turbulent round free jet...
  - round nozzle with a diameter $D = 4$ mm
  - nozzle exit velocity $U_J \simeq 7$ m/s

  $\Rightarrow Re_D = U_J D/\nu \simeq 2.8 \times 10^4$
- ... seeded with tracers...
  - neutrally buoyant spherical polystyrene tracers with a diameter $d_p = 250$ µm
  - specific seeding only through the nozzle

  $\Rightarrow$ **nozzle seeding**
- ... tracked by cameras.
  - three high-speed cameras oriented orthogonal to the brown faces (icosahedral geometry) in a back light configuration
  - measurement volume spanning 100 mm $\leq z \leq$ 180 mm (jet axis z and $z = 0$ the nozzle exit position)
  - 50 movies of 8000 frames recorded at 6000 fps (inlet valve open some seconds before each recording)

Particle tracking velocimetry



Particle detection → Stereoscopic reconstruction [1, 2] → Tracking

- Cylindrical coordinates (z, r, θ)
- Tracks convolved with a first-order derivative Gaussian kernel $\Rightarrow$ velocity

$\Rightarrow \simeq 10^8$ particle positions and velocities

References

[1] Machicoane *et al.* 2019, *Rev. Sci. Instrum.* **90** (3), 035112.

[2] Bourgoin & Huisman 2020, *Rev. Sci. Instrum.* **91** (8), 085105.

[3] Pope 2000, *Turbulent Flows*, Cambridge University Press.

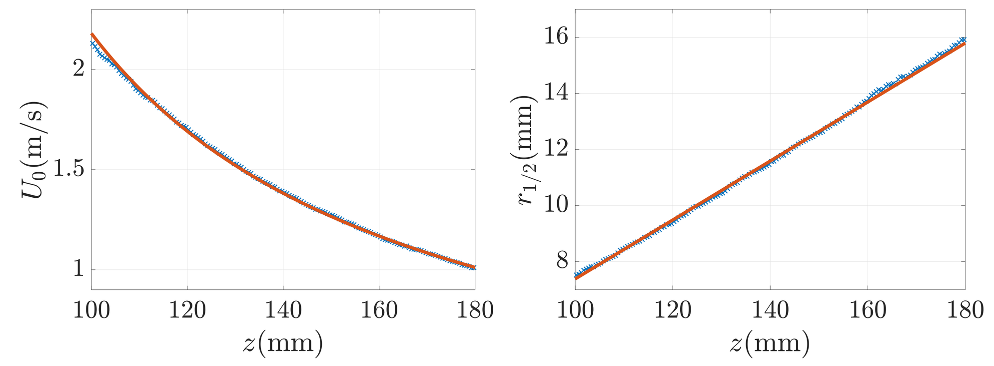
Mean axial velocity $\langle U(z,r)\rangle$

Mean velocity field well-known through Eulerian measurements [3]

- Centerline velocity: $U_0(z) = \langle U(z, r = 0)\rangle$
- Half-width: $\langle U(z, r = r_{1/2}(z))\rangle = \frac{1}{2}U_0(z)$

In the self-similar region (for $z \gtrsim 15D = 60$ mm)

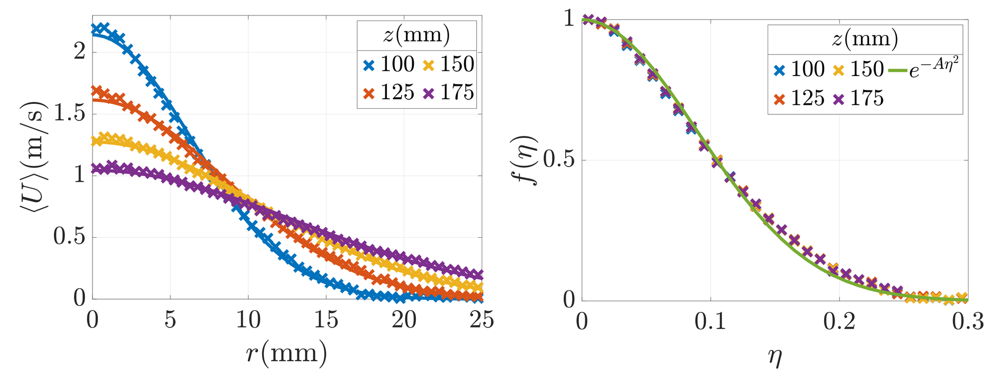
$$\frac{U_0(z)}{U_J} = \frac{B}{(z - z_0)/D}$$
$$r_{1/2}(z) = S(z - z_0)$$



$\Rightarrow$ Experimental results: $z_0 \simeq 4D$, $B = 5.3$ and $S = 0.105$

Self-similar radial profile: $f(\eta) = \langle U(z,r)\rangle / U_0(z)$ with $\eta = r/(z - z_0)$

$$f(\eta) = e^{-A\eta^2}$$
with $A = \log(2)/S^2$



$\Rightarrow$ Same axial velocity field despite the nozzle seeding

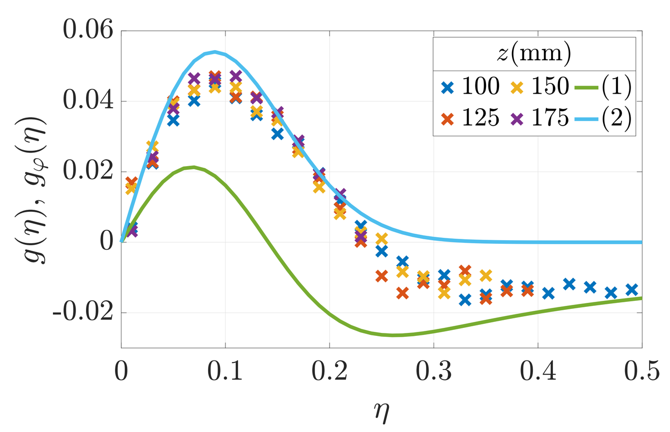
Mean radial velocity $\langle V(z,r)\rangle$ (1/2)

Self-similar radial profile: $g(\eta) = \langle V(z,r)\rangle / U_0(z)$

- Continuity equation for the flow velocity field: $\nabla \cdot \boldsymbol{U} = 0$

$\Rightarrow \eta(\eta f(\eta))' = (\eta g(\eta))'$ [3]

$$\Rightarrow g(\eta) = \eta f(\eta) - \frac{1}{\eta}\int_0^{\eta} x f(x)\, \mathrm{d}x \quad (1)$$



Incorrect function (1): flow velocity field $\boldsymbol{U} \neq \boldsymbol{U}_\varphi$ tracer velocity field

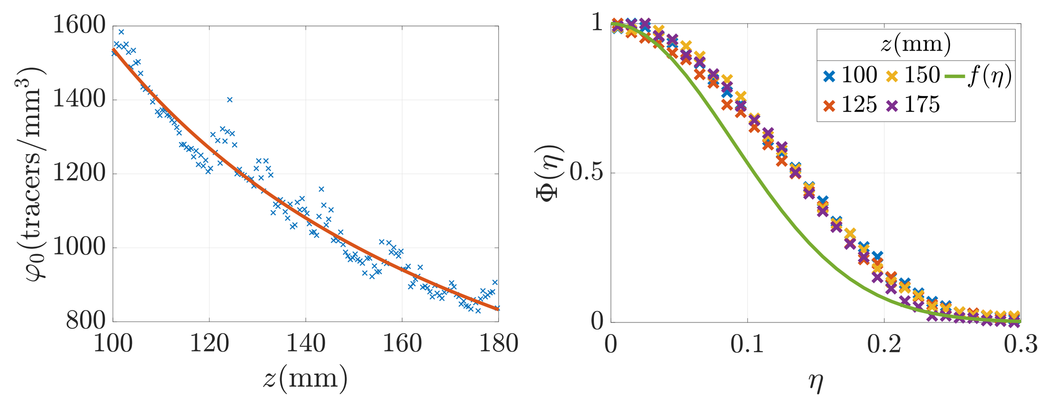
$\Rightarrow$ Entrained flow not tagged by the tracers due to the nozzle seeding (higher measured radial velocity): influence of entrainment up to the core of the jet

Mean radial velocity $\langle V(z,r)\rangle$ (2/2)

- Mean tracer density: $\varphi(z, r)$ with $\varphi_0(z) = \varphi(z, r = 0)$

Self-similar radial profile: $\Phi(\eta) = \varphi(z,r)/\varphi_0(z)$

$$\phi_0(z) \propto \frac{1}{z - z_0}$$



$\Rightarrow$ Same behavior for $\Phi(\eta)$ and $f(\eta)$ (except wider $\Phi(\eta)$)

- Continuity equation for the tracer velocity field: $\nabla \cdot (\varphi\, \boldsymbol{U}_\varphi) = 0$

$\Rightarrow g_\varphi(\eta) = \eta f_\varphi(\eta)$ (2) with $f_\varphi(\eta) \simeq f(\eta)$ from experiments

Correct function (2): smaller maximum and negative part for $\eta \gtrsim 0.2$ due to some entrained tracers remaining in the tank

$\Rightarrow$ A new model coherent with experimental results

$$\Rightarrow g(\eta) = g_\varphi(\eta) - \underbrace{\frac{1}{\eta}\int_0^{\eta} x f(x)\, \mathrm{d}x}_{\text{entrainment term}} \quad (3)$$

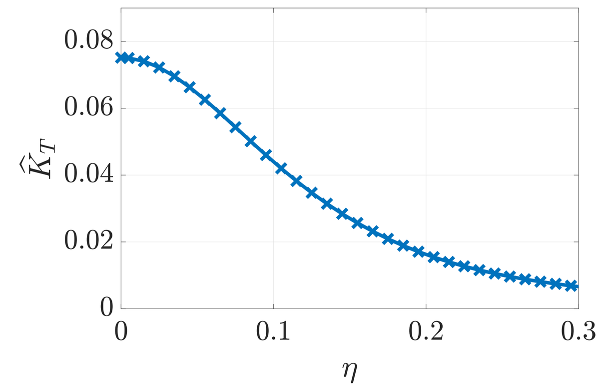
A diffusive model

Advection-diffusion equation: $\nabla \cdot (\varphi\, \boldsymbol{U} - K_T \nabla \varphi) = 0$

with K_T the turbulent diffusion coefficient, $\widehat{K}_T = K_T/(U_0 r_{1/2})$ normalized

To be coherent with (3) $\Rightarrow \widehat{K}_T(\eta) = -\frac{1}{S}\frac{\Phi(\eta)}{\Phi'(\eta)}\frac{1}{\eta}\int_0^{\eta} x f(x)\, \mathrm{d}x$

$\Rightarrow$ Link K_T to the entrainment term and the "compressibility" $\Phi'(\eta)/\Phi(\eta)$



K_T linked to the known turbulent viscosity ν_T through the turbulent Prandtl number $\sigma_T = \nu_T/K_T \simeq 0.6$

$\Rightarrow$ Coherent with scalar transport values [3]

Conclusion

Lagrangian tracer velocity field $\nabla \cdot \boldsymbol{U}_\varphi \neq 0$ due to the nozzle seeding but $\nabla \cdot (\varphi\, \boldsymbol{U}_\varphi) = 0$ and $\nabla \cdot (\varphi\, \boldsymbol{U} - K_T \nabla \varphi) = 0$ with the new turbulent diffusion coefficient K_T linked with entrainment

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