Intermittency of velocity circulation in quantum turbulence

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Introduction

Superfluids

Properties

1. Fluids with no viscosity
2. Vortex filaments with quantised circulation (quantum vortices)

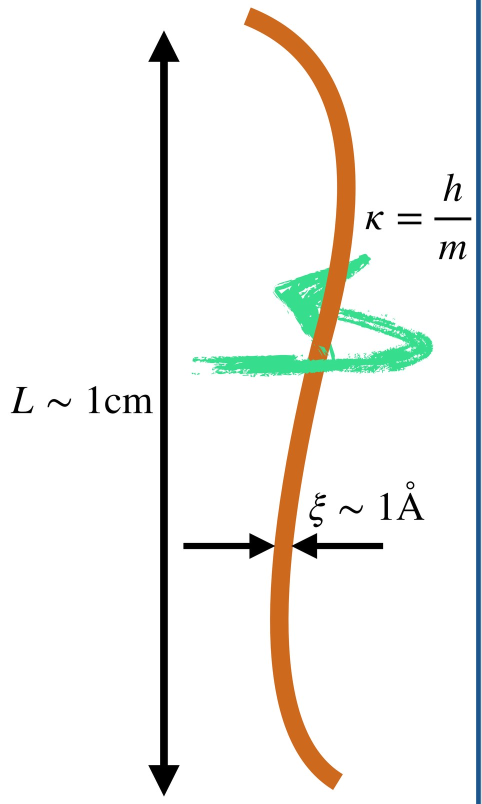
Examples

1. Liquid Helium (T<2.17 K)
2. Bose Einstein condensates (T~100 nK)
3. Light in nonlinear media (T~300 K)
4. Core of neutron stars (T~10⁹ K)

- The vorticity field is concentrated along vortex lines with quantized circulation

$$\Gamma = \oint_{\mathscr{C}} \mathbf{v} \cdot d\mathbf{l} = n\kappa \qquad n \in \mathbb{Z}$$

- Statistics of velocity circulation in classical turbulence is less intermittent than velocity increments. *(Iyer et. al. 2019)*
- Intermittency: Dynamics of extreme events / Deviations from Kolmogorov theory. *(Frisch 1995)*
- The discrete nature of circulation in superfluids may help us to better understand the dynamics of classical fluids.



Model and methods

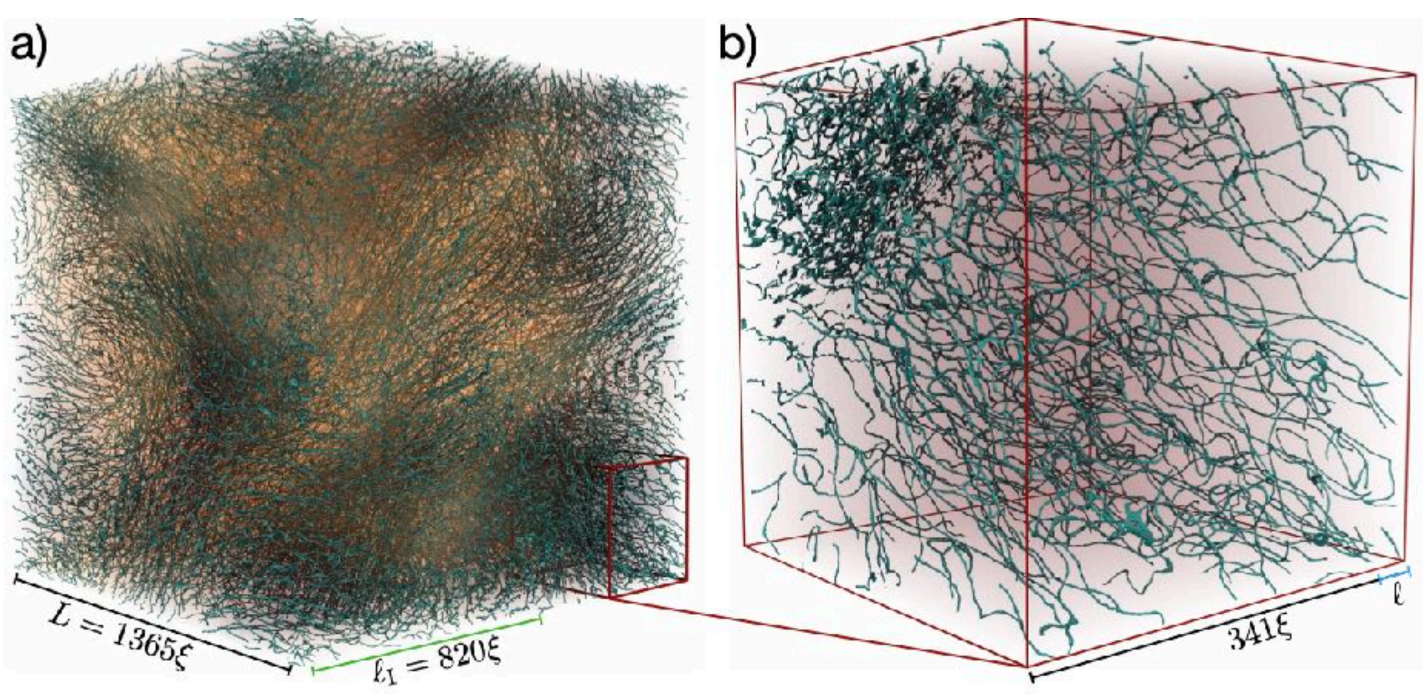
- Superfluids at T~0 can be described by the Gross-Pitaevskii (GP) equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi - \mu\psi$$

- The Madelung transformation provides a hydrodynamic description

$$\psi = \sqrt{\frac{\rho}{m}}e^{im\phi/\hbar} \qquad \rho = m|\psi|^2 \rightarrow \text{Fluid density} \qquad \mathbf{v} = \nabla\phi \rightarrow \text{Fluid velocity}$$

DNS with 2048³ grid points of a decaying ABC flow.

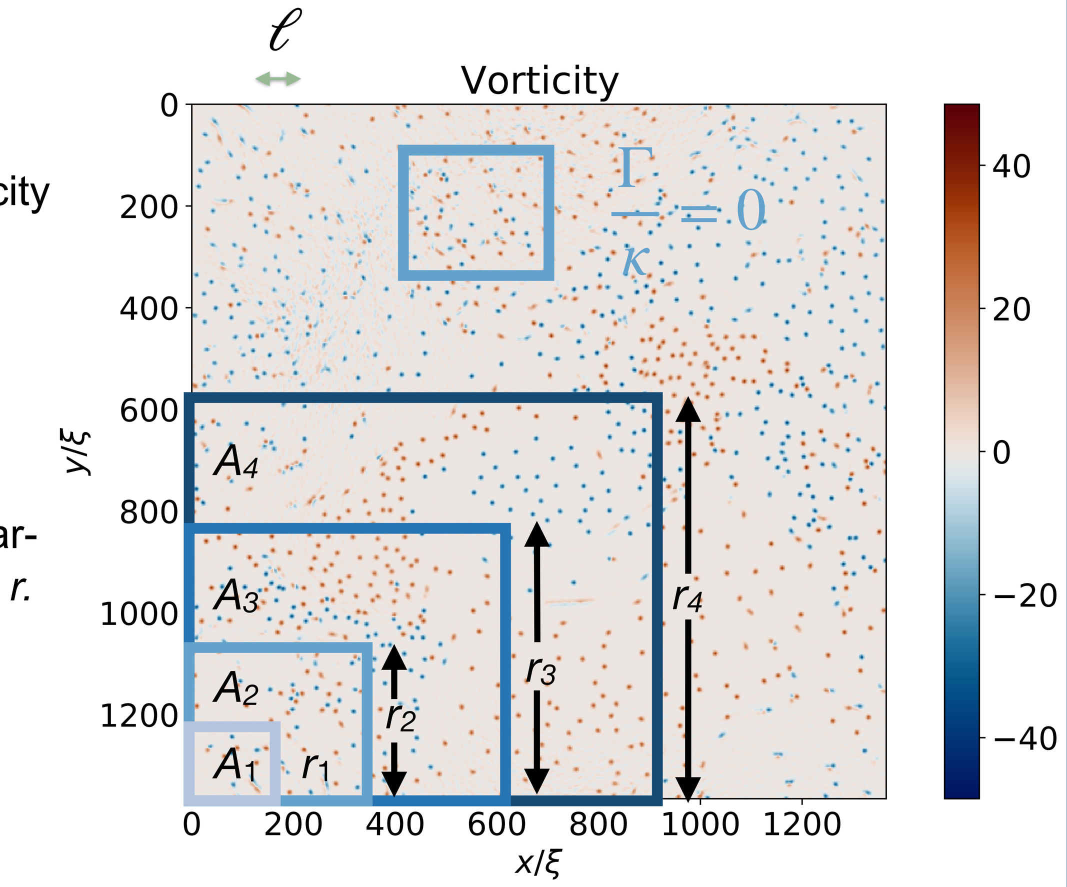


Circulation statistics

- Visualization of the regularized vorticity over a 2D slice of the box.

$$\boldsymbol{\omega} = \nabla \times \sqrt{\rho}\mathbf{v}$$

- Study of the full 3D system with planar-squared loops of different linear sizes r.
- Observation of vortex polarization.



Probability of finding at least one vortex:

$$\beta_r = (r/\ell)^2$$

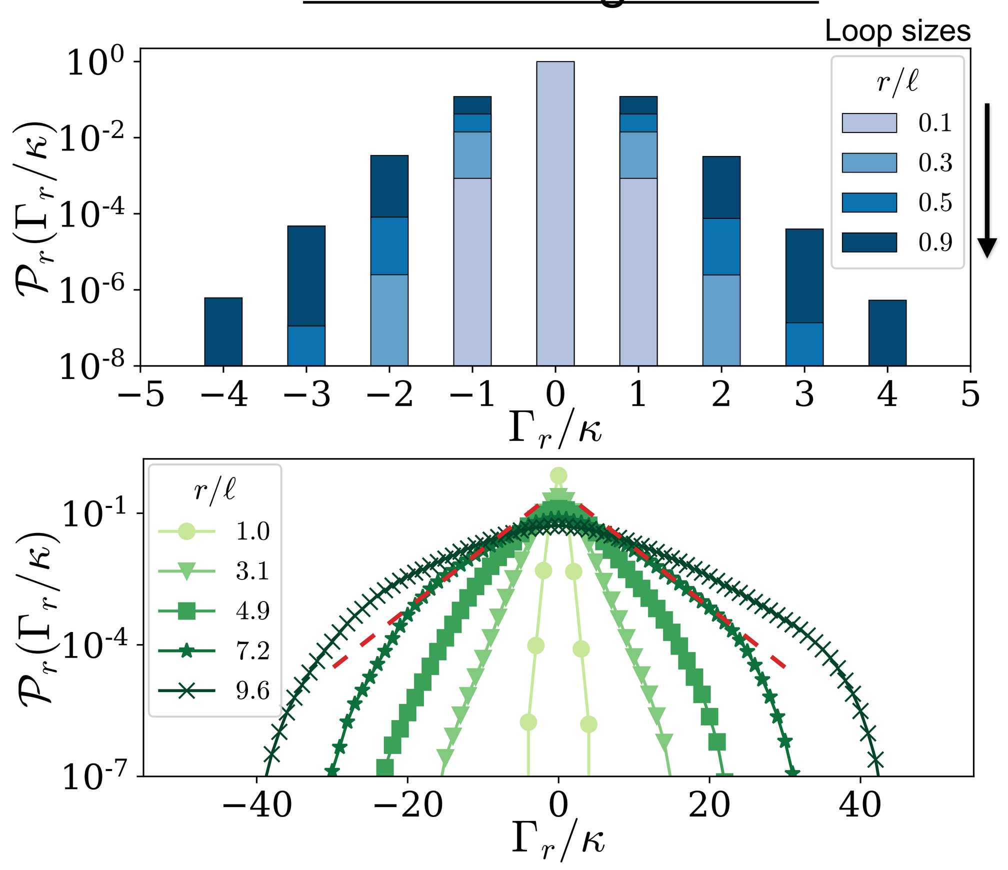
Small scales

High probability of finding no vortices

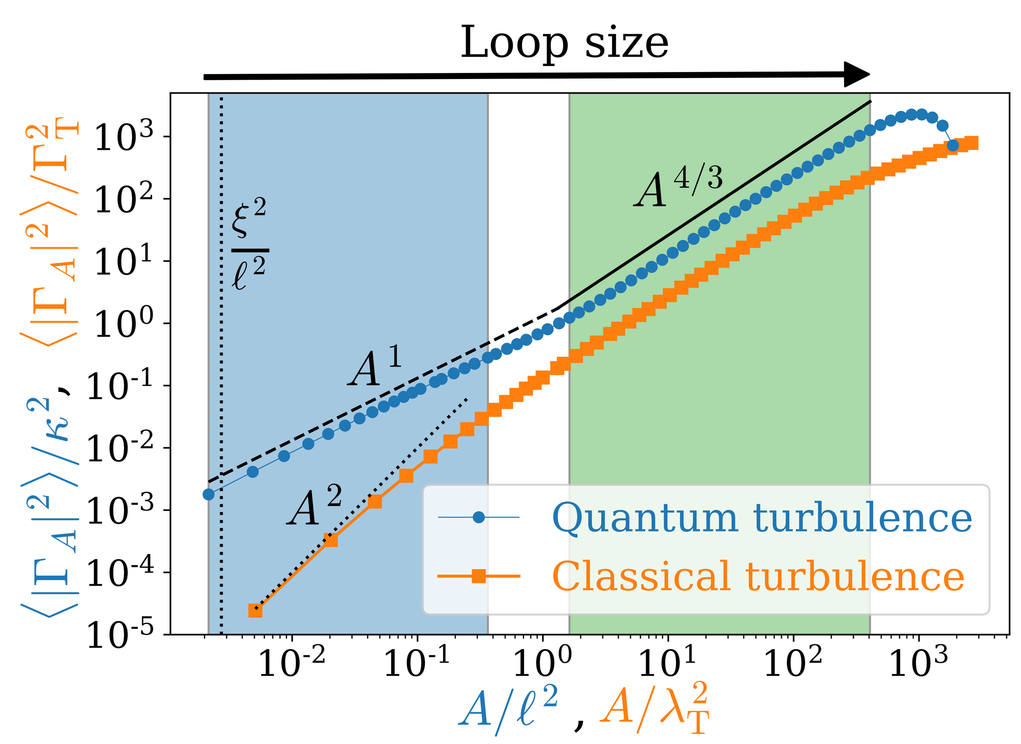
Large scales

Power law behaviour close to the core

Probability distributions at small and large scales



Circulation variance



- Kolmogorov scaling for the circulation variance:

$$\langle\Gamma_A^2\rangle = C_2\epsilon^{2/3}A^{4/3}$$

- Taylor microscale

$$\lambda_T = \frac{v_{rms}}{\sqrt{\langle(\partial_x v_x)^2\rangle}}$$

$$\langle\Gamma_A^2\rangle \approx \frac{1}{3}\langle|\omega|^2\rangle A^2$$

Intermittency of velocity circulation

Moments of circulation

Kolmogorov scaling for circulation

Large scales

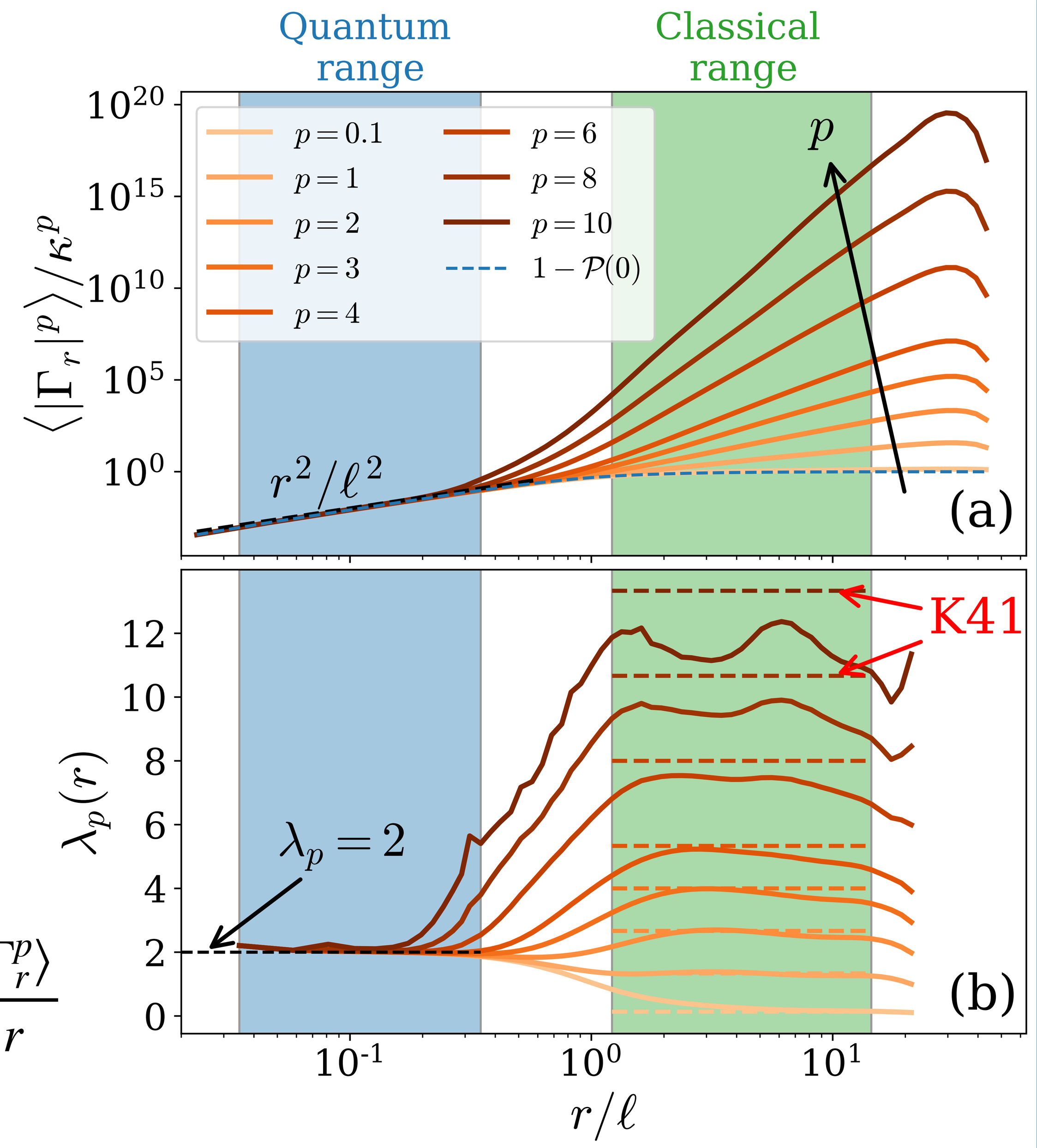
$$\langle\Gamma_r^p\rangle = C_p\epsilon^{p/3}r^{4p/3}$$

Small scales

$$\frac{\langle\Gamma_r^p\rangle}{\kappa^p} \sim \frac{r^2}{\ell^2}$$

Local scaling exponent

$$\lambda_p(r) = \frac{d\log\langle\Gamma_r^p\rangle}{d\log r}$$



Scaling exponents

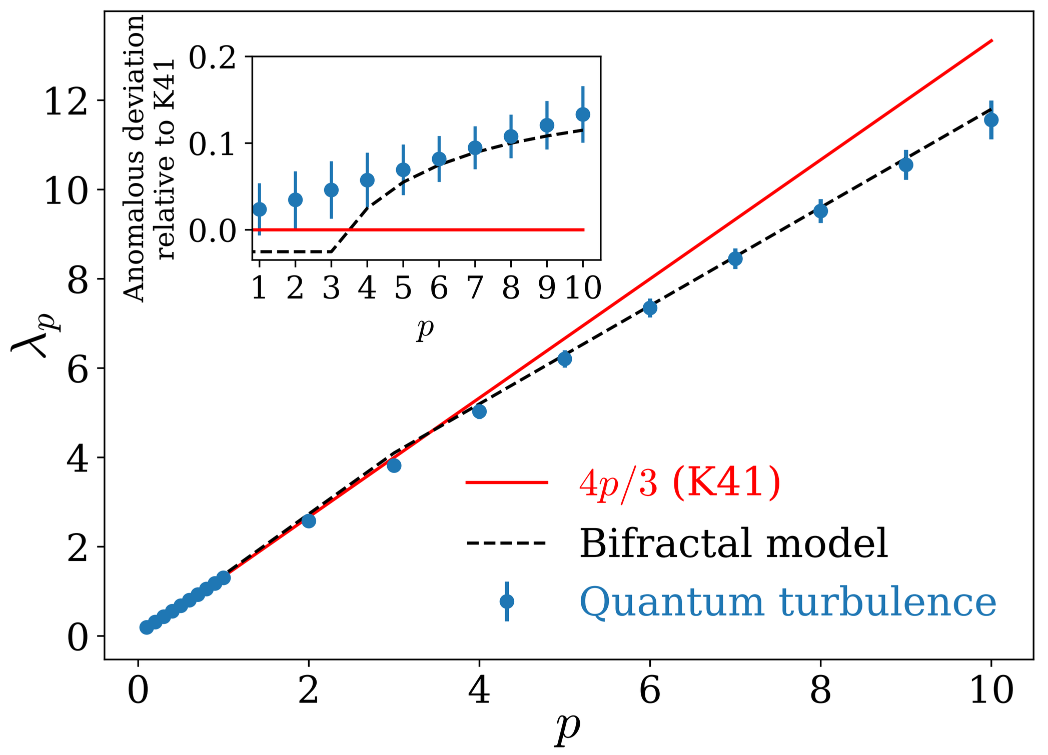
$$\frac{\langle\Gamma_r^p\rangle}{\kappa^p} \sim \left(\frac{r}{\ell}\right)^{\lambda_p}$$

Bifractal model

$$\lambda_p = 4p/3 \qquad p < 3$$

$$\lambda_p = 1.1p + 0.8 \qquad p > 3$$

- Deviations from K41 at p>3, but smaller than in velocity increments.
- Behaviour similar to the classical case.



Conclusions and perspectives

- Velocity circulation in quantum turbulence at large scales follows the same dynamics as classical turbulence.
- Does the polarization contribute to the intermittency?
- How does the spatial distribution of vortices contribute to the intermittency?

References

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