Capillary surfers on a vibrating bath

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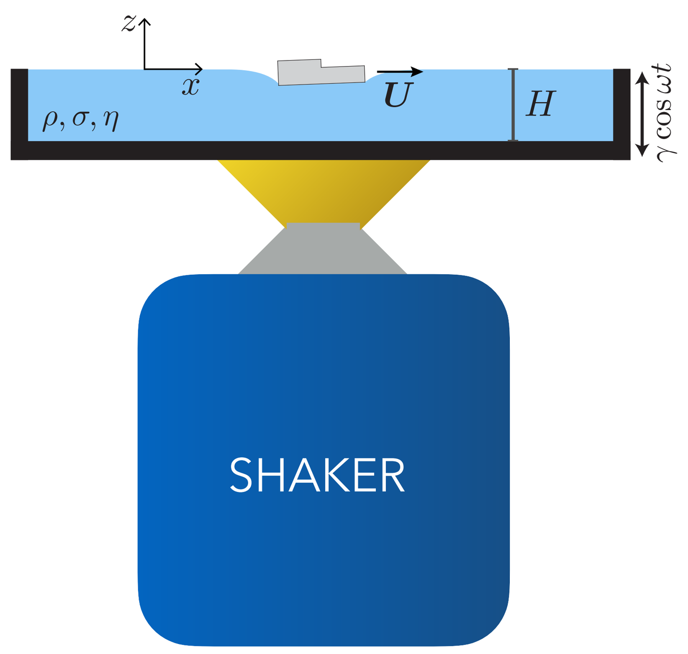
AUO acknowledges support from the Simons Foundation (Collaboration Grant for Mathematicians, Award No. 587006). GP acknowledges the CNRS Momentum program for its support. IH and DMH acknowledge the support of the Brown Undergraduate Teaching and Research Award.

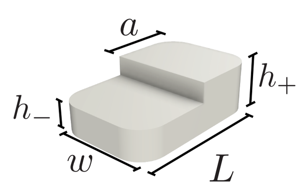
1. Experimental setup

We introduce ***capillary surfers***: solid particles propelled by their self-generated wave field on the surface of a vibrating liquid bath.



ρ density | σ surface tension | η viscosity

In our experiments, surfers were made of PTFE and had the shape shown on the right:



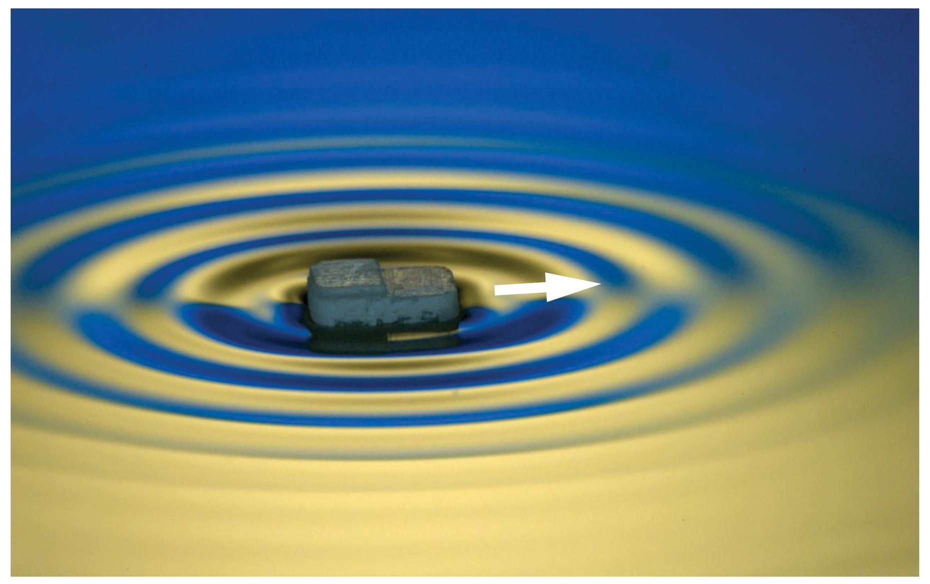
Surfers were deposited on a bath of water-glycerol mixture and were supported at the liquid-air interface by virtue of the equilibrium between their weight, hydrostatic forces and surface tension.

As a result of their mass asymmetry, surfers were slightly tilted in equilibrium.

The liquid bath was vertically driven by an electromagnetic shaker with acceleration

$$\Gamma(t) = \gamma\cos(2\pi f t)$$

As soon as the bath was set into vibration, a surfer generated **propagating surface waves** as a result of the relative vertical motion between the surfer and bath, a consequence of the surfer's inertia. Correspondingly, the surfer moved with constant speed along its long axis in the direction of its thinner half.

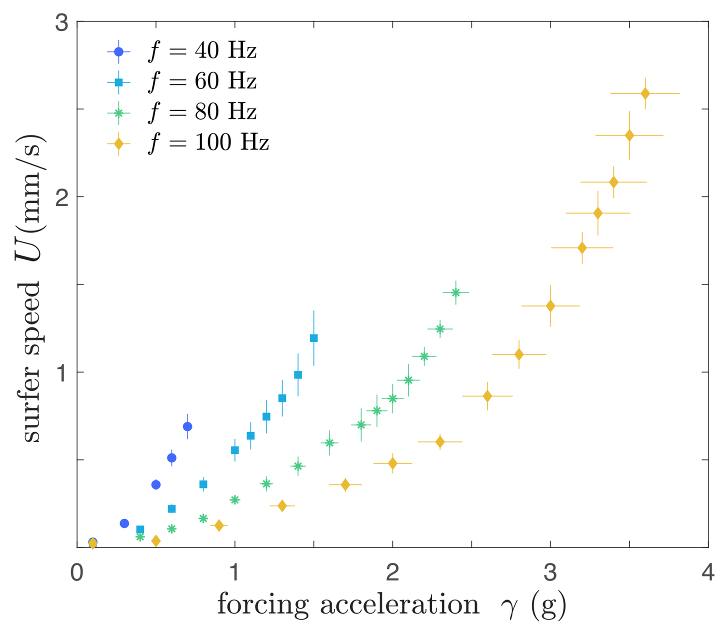


2. Mechanism of self-propulsion

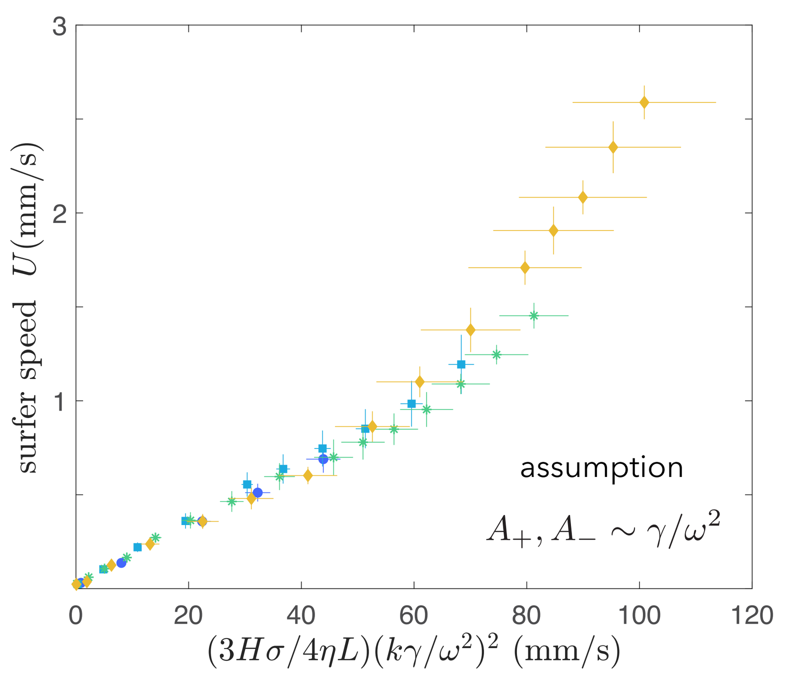
The dependence of surfer's speed on forcing frequency and amplitude is rationalized by considering the **radiation stress S of surface waves**, defined as *"the excess flow of momentum due to the presence of the waves"* (Longuet-Higgins and Stewart, Deep-Sea Research 11, 529 (1964)).

For capillary waves of amplitude A and wavenumber k in the deep-water limit $kH >> 1$

$$S = \frac{3}{4}\sigma A^2 k^2$$



- The surfer exhibits a **fore-aft asymmetry**: waves of larger amplitude A_+ are generated at the stern, where the effective mass is larger, while waves of smaller amplitude A_- are generated at the bow.



The surfer thus experiences a net propulsive force

$$F_p = \frac{3}{4}\sigma k^2 w (A_+^2 - A_-^2)$$

that is resisted by viscous shear stress

$$F_D \approx \eta w L U/H$$

Balancing these forces $F_p \sim F_D$

we obtain $$U \approx \frac{3H\sigma k^2}{4\eta L}\left(A_+^2 - A_-^2\right)$$

3. Two-surfer interactions

Surfers interact with each other through the propagating waves they generate on the fluid interface. Two surfers of equal size and speed can arrange in up to seven **modes of interaction**.

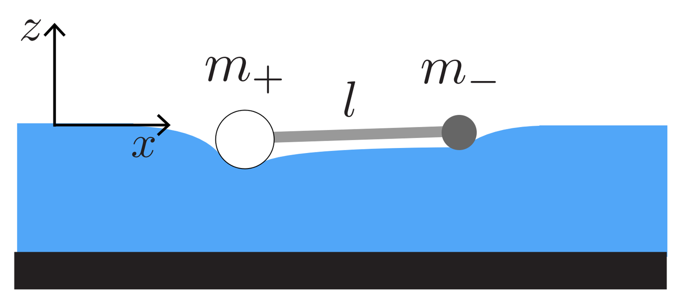
We model a surfer of mass m as a pair of point masses m_+ and m_- with

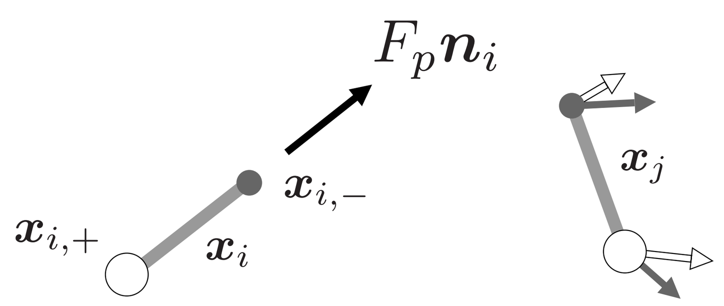
$$m_+ + m_- = m,$$

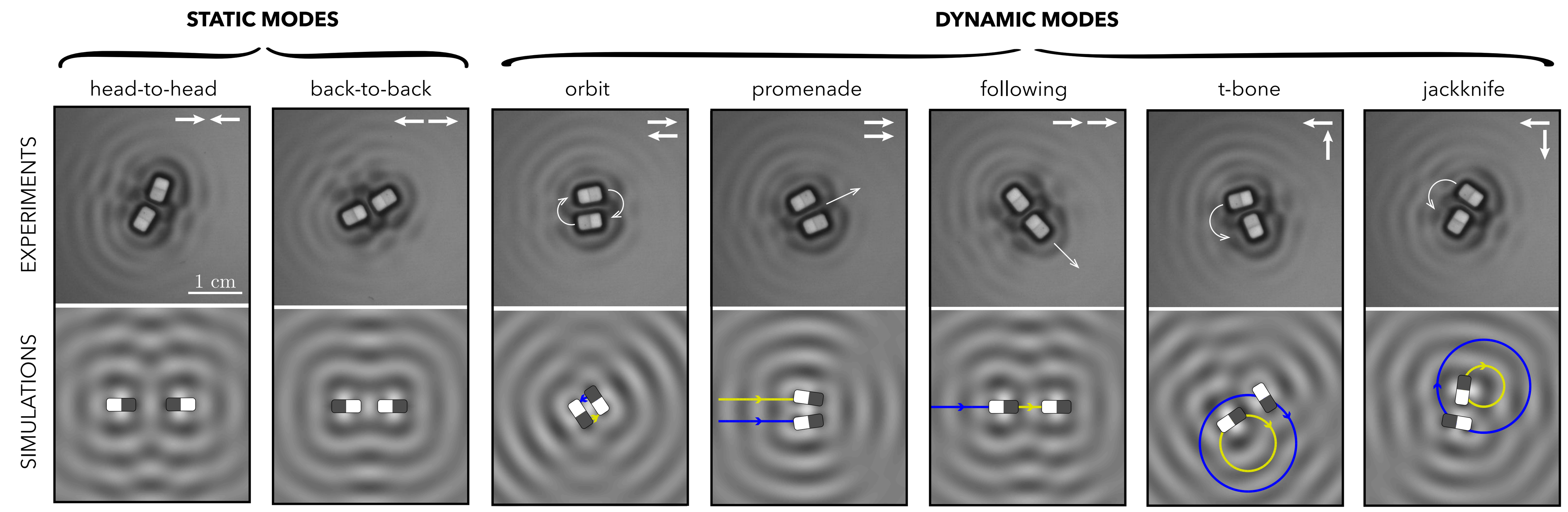
connected by a massless rod of length

$$l = L/2$$

The masses are assumed to oscillate vertically at the forcing frequency and amplitude of the bath, and thus generate capillary waves.







Since the waves are of small amplitude, their form may be deduced by solving the linearized hydrodynamic problem of a periodically oscillating point force acting on a fluid interface (De Corato and Garbin, J. Fluid Mech. 847, 71 (2018)).

Both the surfers and capillary waves oscillate at the forcing frequency, and the capillary interaction force is nonzero when time-averaged over the forcing period. This force has an oscillatory spatial dependence on the capillary wavelength $\Phi(kr)$.

The **equations of motion** for the *ith* surfer write

$$m\ddot{\boldsymbol{x}}_i + \frac{m}{\tau_v}\dot{\boldsymbol{x}}_i = F_p\boldsymbol{n}_i + F_c\sum_{\alpha,\beta=\pm1}\mu_\alpha\mu_\beta\sum_{j\neq i}\Phi(k|\boldsymbol{x}_{i,\alpha}^{j,\beta}|)\hat{\boldsymbol{x}}_{i,\alpha}^{j,\beta},$$

$$I\ddot{\theta}_i + \frac{I}{\tau_v}\dot{\theta}_i = -lF_c\sum_{\alpha,\beta=\pm1}\mu_\alpha\mu_\beta\sum_{j\neq i}\alpha\mu_{-\alpha}\Phi(k|\boldsymbol{x}_{i,\alpha}^{j,\beta}|)\boldsymbol{n}_i\times\hat{\boldsymbol{x}}_{i,\alpha}^{j,\beta}$$

propulsive force (radiation stress); capillary wave force; inertia; viscous drag; sum over masses; sum over surfers; capillary wave torque

where

$\boldsymbol{x}_{i,\alpha} = \boldsymbol{x}_i - \alpha\mu_{-\alpha} l\boldsymbol{n}_i$

$\boldsymbol{x}_{i,\alpha}^{j,\beta} = \boldsymbol{x}_{j,\beta} - \boldsymbol{x}_{i,\alpha}$

$\hat{\boldsymbol{x}} = \boldsymbol{x}/|\boldsymbol{x}|$

I moment of inertia

$F_c = (mg)^2 k/\sigma$ capillary force coefficient

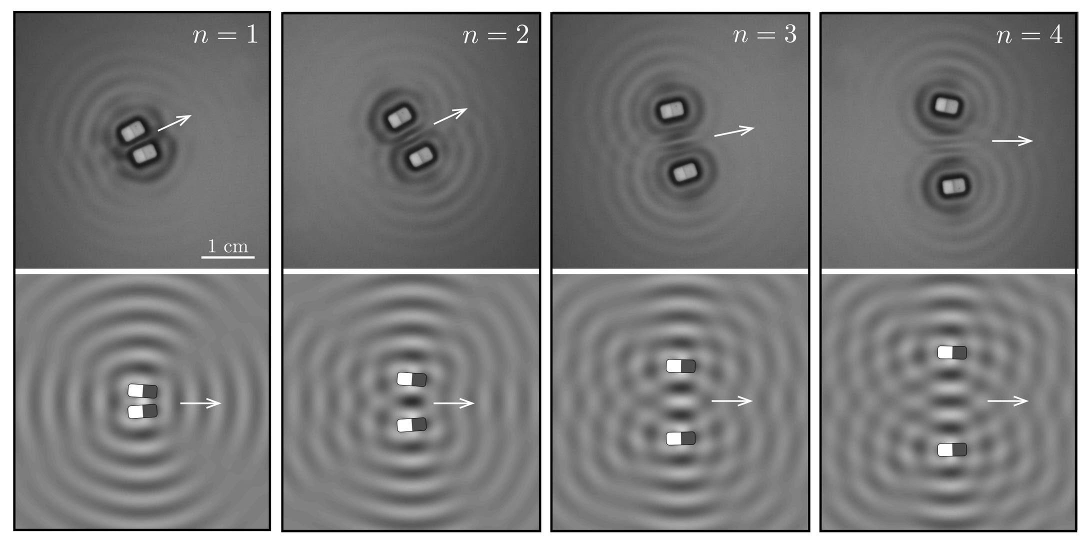
$\tau_v = mH/\eta w L$ viscous timescale

4. Multistability

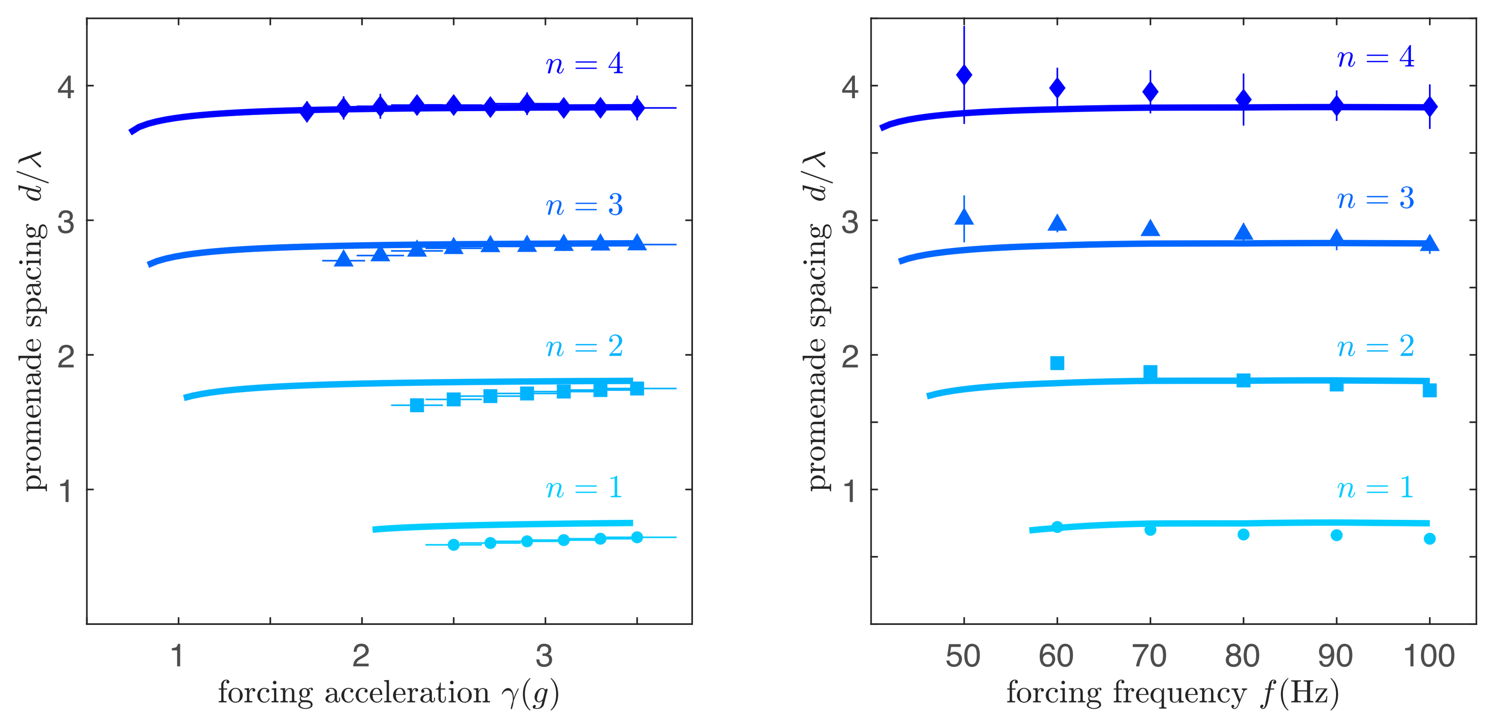
In each mode the two surfers exhibit discrete equilibrium spacings

$$d \approx n\lambda \quad n = 1, 2, 3, \ldots$$

λ capillary wavelength



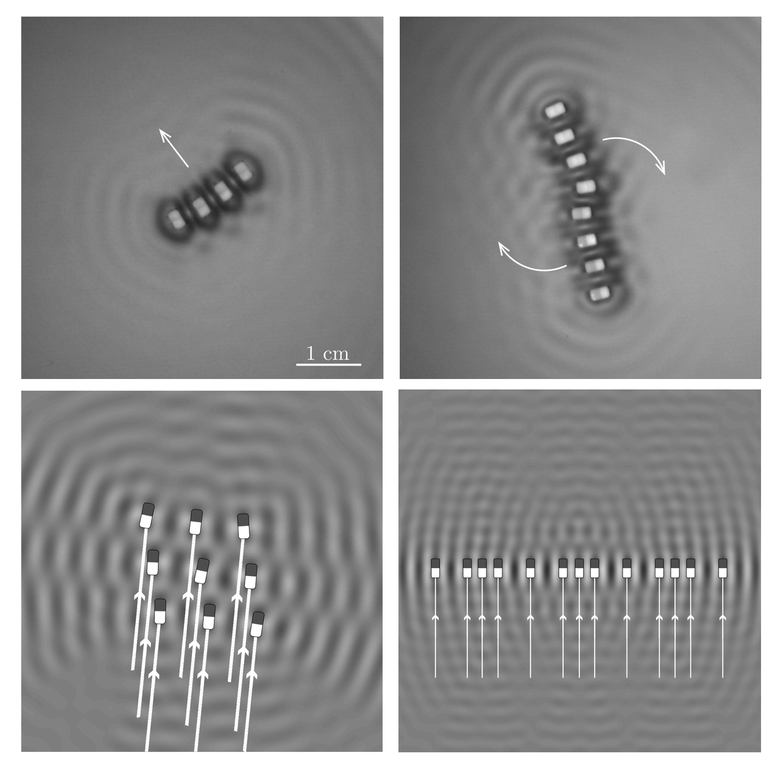
The equilibrium spacings are quantized on the capillary wavelength in the range of forcing frequencies and amplitudes explored in experiments (points below) and captured by the theoretical model (lines below).



5. A novel active system

Many-body experiments and simulations show that capillary surfers have potential to serve as constituents of a novel active system.

While there have been extensive studies on overdamped active systems (*i.e.* bacterial suspensions) mediated by viscous hydrodynamic forces that decay monotonically with distance, a collection of surfers has the peculiar feature of being characterised by **wave-mediated interactions**, which results in long-range **spatially-oscillatory forces** interaction laws defined by alternating regions of attraction and repulsion. This feature is a consequence of fluid inertia, and responsible for the **multistability** of a discrete set of interaction states.



Ultimately, the surfer system is a uniquely tunable and accessible experimental platform that has the potential to fill the gap between active systems at low and high Reynolds numbers: surfers self-propel at **intermediate Reynolds** number

$$Re = \frac{\rho U L}{\eta} \approx 1 - 10$$

where both inertia and viscous forces are relevant.

I. Ho, G. Pucci, A. U. Oza and D. M. Harris. Capillary surfers: wave-driven particles at a fluid interface. arXiv:2102.11694 (2021).