The linear instability of the stratified plane Poiseuille flow

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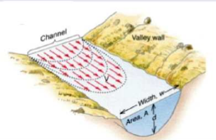
Abstract: We present here the stability analysis of a plane Poiseuille flow stably stratified in density along the vertical direction, i.e. orthogonal to the horizontal shear. Density stratification is ubiquitous in nature and we may think to water flows in submarine canyons, to winds in deep valleys or to laminar flows in rivers or canals where stratification can be due to temperature or salinity gradients. Our study is based on laboratory experiments, on a linear stability analysis and on direct numerical simulations. It is shown that the instability belongs to a class of instabilities caused by the resonant interaction of Doppler shifted internal gravity waves and Tollmien Schlichting waves. The comparison of experimental data with the theoretical threshold and the critical wavenumbers calculated by linear analysis is excellent although at different Schmidt numbers . Finally, direct numerical simulations permit to complete the description of this new instability for the Plane Poiseuille flow.

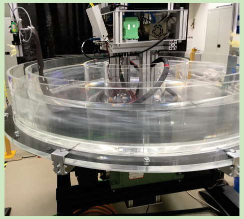
Flows in submarine canyons or in rivers and canals, winds in deep valleys where stratication can be due to temperature or salinity gradients



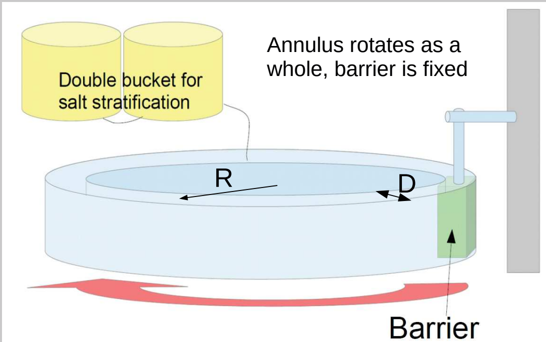
Stability of Poiseuille flow subject to stratification ?

Instability Pattern : Layering and meandering

The flow is driven by the walls of a slowly rotating cylindrical annulus. The gap D is equal to 8.5 cm and the radius of the inner wall is R= 0,715 m. A barrier is placed in the fluid to block the flow and create a counter flow. The result is a 2D Plane Poiseuille flow in the bulk.

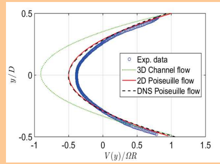


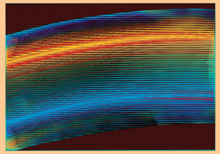
Not straitforward to create a stratified Poiseuille Flow in the Lab !

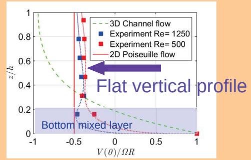


Experimental device : Rotating Annulus with fixed barrier

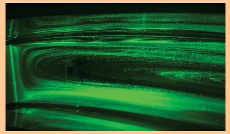
Base flow PIV velocity profiles : Because of mixing between bottom and barrier : field is close to a 2D plane Poiseuille flow







Vorticity field and streamlines in the horizontal plane



Visualisation of the flow near the barrier

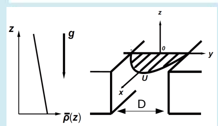
Brunt-Väisälä frequency N

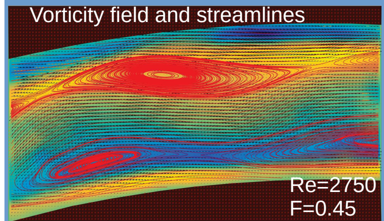
Reynolds number= 3 Ω RD/ 4ν

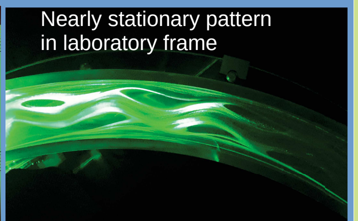
Froude number= 3 Ω R/ ND

Schmidt number = 700 (salt)

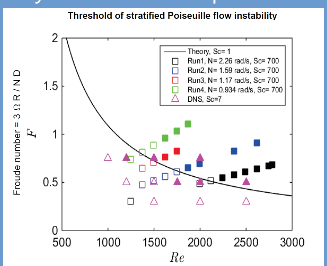
$V(y) = \Omega R/2\ [12\ (y/D)^2-1]$

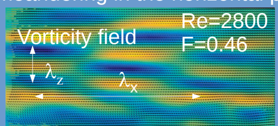


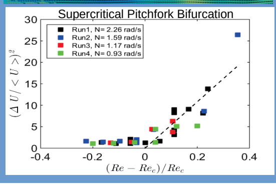




PIV measurement and visualisation of a meandering in the horizontal plane and Layers in the vertical plane

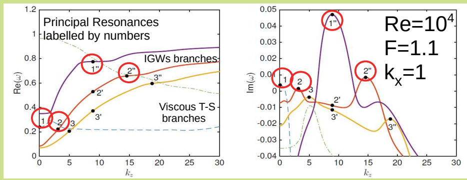




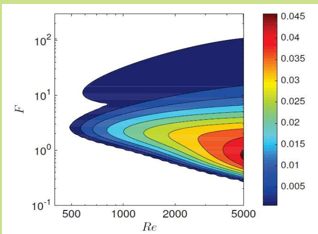


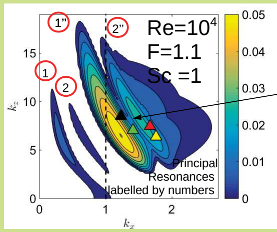
Linear Stability Analysis (Schmidt number = 1)

Mechanism : Resonances of Doppler shifted waves



Frequency ω_r and growth rate ω_i versus vertical wave number k_z





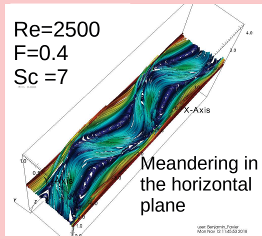
Experimental measures

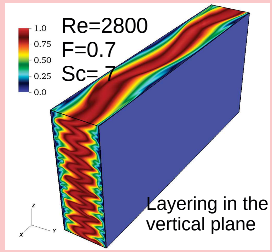
Theoretical contour of growth rate ω_i in the (F, Re) plane and in the $(k_x\ k_z)$ plane with experimental results

Direct Numerical Simulation - Nek5000
Spectral elements - Periodic in vertical and streamwise directions

Resonance of Doppler shifted viscous T.S. and gravity waves

Saturated state





DNS of Base flow : Error versus Poiseuille Parabolic profile
Aspect ratio = 50 - Re=2500