Intermittency in a turbulent model a consequence of stationary constraints

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Abstract :

We explored some new constraints that stationary processes impose on the fluctuations of power in the context of turbulence. Here we first recall some properties of the fluctuations of the injected power, the dissipated power and the energy flux that have to converge at vanishing frequency. Then we show that these properties are fulfilled by GOY–shell model that share intermittent properties of turbulence. Hence constraints on the power fluctuations might force some intermittency in the GOY–shell model . Indeed we show that the constraint on the power fluctuations implies a relation between scaling exponents. This relation is fulfilled by the GOY–shell model and agrees the She-Leveque formula. It also fixes the intermittent parameter of the log-normal model to a realistic value.

Properties of dissipative and turbulent systems

Dissipative systems: $\frac{dE}{dt} = I - D$: Energy balance with $I \equiv$ injected and $D \equiv$ dissipated powers

Stationary processes $\Rightarrow \langle I \rangle = \langle D \rangle$ but also the equivalent relations :

(1) $\lim_{\omega\to 0} |\hat{I}(\omega)|^2 = \lim_{\omega\to 0} |\hat{D}(\omega)|^2$ with $\hat{X}(\omega)$the Fourier Transform of X

(2) $\int_0^\infty \langle \Delta I(t)\cdot \Delta I(t+\tau)\rangle\, d\tau = \int_0^\infty \langle \Delta D(t)\cdot \Delta D(t+\tau)\rangle\, d\tau$

with $\Delta X(t) = X(t) - \langle X \rangle$

(3) $\tau_I \sigma(I)^2 = \tau_D \sigma(D)^2$ with $\sigma(X)^2 = \langle \Delta X^2 \rangle$

and $\tau_X(t) = \frac{1}{\sigma(X)^2}\int_0^\infty \langle \Delta X(t)\cdot \Delta X(t+\tau)\rangle\, d\tau$

Turbulent systems: dissipation occurs at much smaller scales than injection. The input power must cascade up to the dissipative scale.

$\frac{dE_K}{dt} = I - \Pi_K$ where E_K= coarse-grain energy of the velocity Low Pass filtered up to the wave number K;

Π_K= energy flux up to K

Turbulent hypothesis: there is an inertial range where Π_K does not depend on the injection and the dissipation. In addition to $\langle I \rangle = \langle \Pi_K \rangle = \langle D \rangle$ one has

$$\tau_I \sigma(I)^2 = \tau_{\Pi_K} \sigma(\Pi_K)^2 = \tau_D \sigma(D)^2$$

Scaling exponents and intemittency

Scaling exponents for the Fourier mode:

$$\langle u_k^p \rangle \propto k^{-\zeta_p}$$

K41: $\langle I \rangle = \langle \Pi_K \rangle \Longrightarrow \zeta_3 = 1$

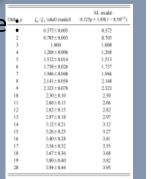
Intermittency $\Longrightarrow \zeta_p \neq$ p/3

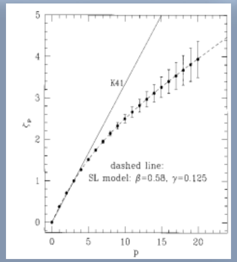
- Log-normal model:

$\zeta_p = \frac{p}{3} + \mu/18(3p - p^2)$

- She-Leveque (SL) model

$$\zeta_p = \gamma p + \frac{(1-3\gamma)\left(1-\beta^{\frac{p}{3}}\right)}{1-\beta};\ \gamma = \frac{1}{8};\ \beta \sim 0.58$$





[Leveque and She PRE 55-3 (1997)]

The GOY-shell model

Dynamical equation mimicking Navier-Stokes equation in discrete and exponentially, $k_n = k_o 2^n$ spaced wave number space. It satisfies the SL model

$$\frac{du_n}{dt} = \mathcal{F}(u_{n-2}, u_{n-1}, u_{n+1}, u_{n+2}) + f_4 \cdot \delta_{n,4} + \nu k_n^2 u_n\,;$$

$u_n \in \mathbb{C}$; $n \in [1:N]$, $k_n = 2^n/16$

$$\mathcal{F}(u_{n-2}, u_{n-1}, u_{n+1}, u_{n+2}) = ik_n\left(u_{n+1}u_{n+2} - \frac{u_{n+1}u_{n-1}}{4} - \frac{u_{n-1}u_{n-2}}{8}\right)^* + cc$$

In the inertial range: $\frac{dE_K}{dt} = I_K - \Pi_K$ $\quad E_K = \sum_{n=1}^{M<N} |u_n|^2/2$;

$I_K = I = f_4 \cdot u_4^* + cc$;

$\Pi_K = i\,K(u_{M+2}u_{M+1}u_M + u_{M+1}u_M u_{M-1}/4)^*$

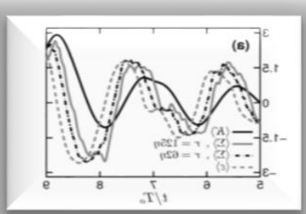
$4 < M \ll \left(\frac{I}{\nu^3}\right)^{1/4}$; $K = 2^M/16$

Conclusions

We show that constraints on the fluctuations of energy flux in the GOY shell model imposes a additional relation on the intermittent scaling exponents:

$\zeta_6 - \zeta_2 - 1 = 0$

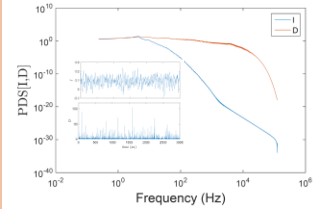
Fitted by the SL model.

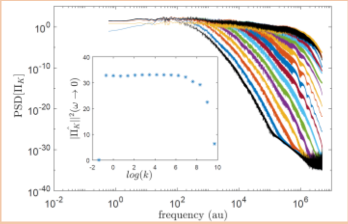


[Cardesa et al PoF **27** 111102]

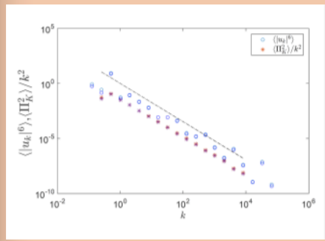
NB: extension to real turbulence not so straightforward because global quantities are averaged in space (those much less intermittent)

Results and scalings





The relation $\tau_I \sigma(I)^2 = \tau_D \sigma(D)^2$
$= \tau_{\Pi_K} \sigma(\Pi_K)^2$holds for the GOY shell model



In the GOY shell model:

$\sigma(\Pi_K)^2 \propto K^2 \langle u^6 \rangle$ (1.a).

$\propto K^{2-\zeta_6}$ (1.b)

$\propto K^{0.27}$ (1.c)

$\Rightarrow \tau_{\Pi_K} \propto K^{-0.27}$

What is $\tau_{\Pi_K} \neq \tau_K (\propto K^{-0.6})$??

τ_K the characteristic time of the shell K

Hypothesis: there is a given characteristic time for coarse-grain quantities. $\Pi_K, U_K = \sum_{i=1}^M u_i$, ...

It can be computed as follow

$$\tau_{\Pi_K} = \frac{\int_0^\infty \langle U_K(0) U_K(\tau)\rangle d\tau}{\sigma(U_K)^2}$$

$$\sim 1/\sigma(U_K)^2 \cdot \sum_{i=1}^M \int_0^\infty \langle u_i(0) u_i(\tau)\rangle\, d\tau$$

$$\sim \frac{\sum_{i=1}^M \tau_i \sigma(u_i)^2}{\sum_{i=1}^M \sigma(u_i)^2} \xrightarrow[K=2^M \gg 1]{} K^{\zeta_2 - 1} \Longrightarrow \tau_{\Pi_K} \propto K^{-0.28}\ !!!!$$

New relation between intermittent scaling exponents

$$\zeta_6 - \zeta_2 - 1 = 0$$

- Holds for She-Leveque Model (K41: $\zeta_5 - \zeta_2 - 1 = 0$)
- Fix the intermittent free parameter μ of the log-normal parameters to a realistic value μ=0.3.