Energy flux between two NESS thermostats.

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Abstract

This experiment allows to investigate in a simplified configuration the relation between energy flux and temperature gradients in generic dissipative driven systems. It consists of two stationary granular gas thermostats excited separately in Non-Equilibrium Steady States (NESS), at distinct temperature, coupled in such a way that energy can flow between them.
This energy flux is characterized for varied granular temperature differences ΔkT. The mean flux is proportional to the temperature gradient, like in the case of equilibrium thermostats. One great advantage of this macroscopic experiment is that the fluctuations of the flux can be precisely measured. The asymmetry of these fluctuations is compatible with the eXchange Fluctuation Theorem (XFT) proposed by Jarzynski and Wójcik in 2004, in the non-dissipative coupling limit. This is particularly surprising as the heat reservoirs are out-of-equilibrium, but promising as a way to address a large class of problems.

1. Experimental setup

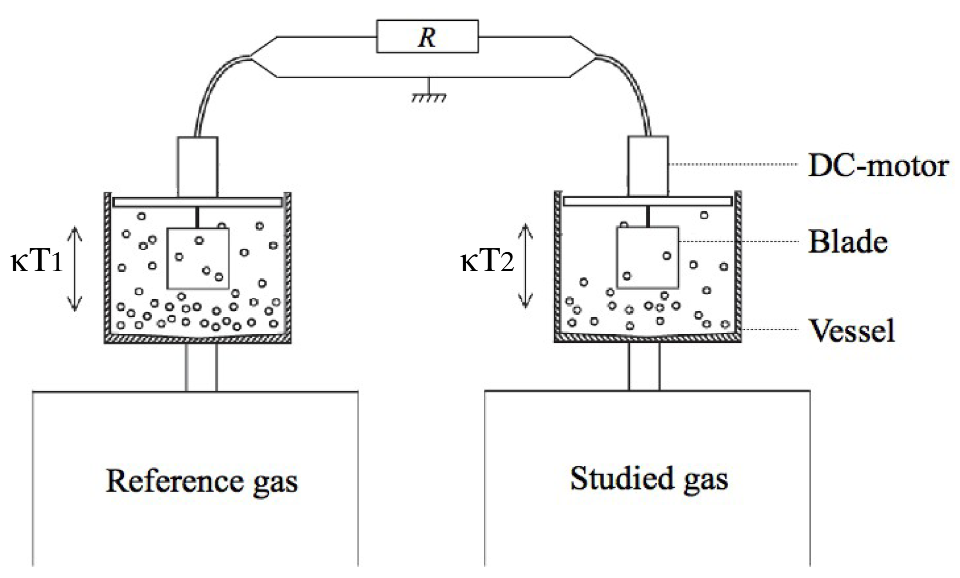
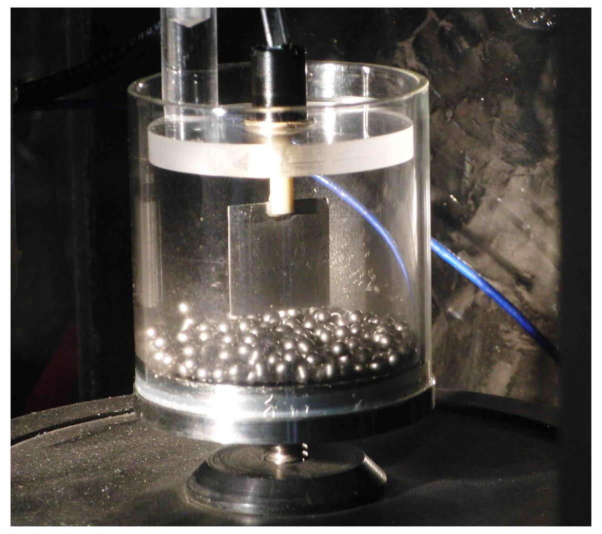


Fig.1: Sketch of the experimental setup: granular gas of ~350 steel beads of 0.1mg, 3mm diameter, accelerated independently at a few g by 2 shakers. 2cm blades are embedded in each gas, free to rotate about a vertical axis. Their rotational motion is detected by reversible DC micro-motors.



Granular gases are nothing but generic Non-Equilibrium Steady State (NESS) heat baths

S/N ratio at cm-scale is such that one can investigate the fluctuations of the flux...

2. Flux and temperatures

A central feature is the reversibility of the DC motors allows energy exchange back and forth between the baths

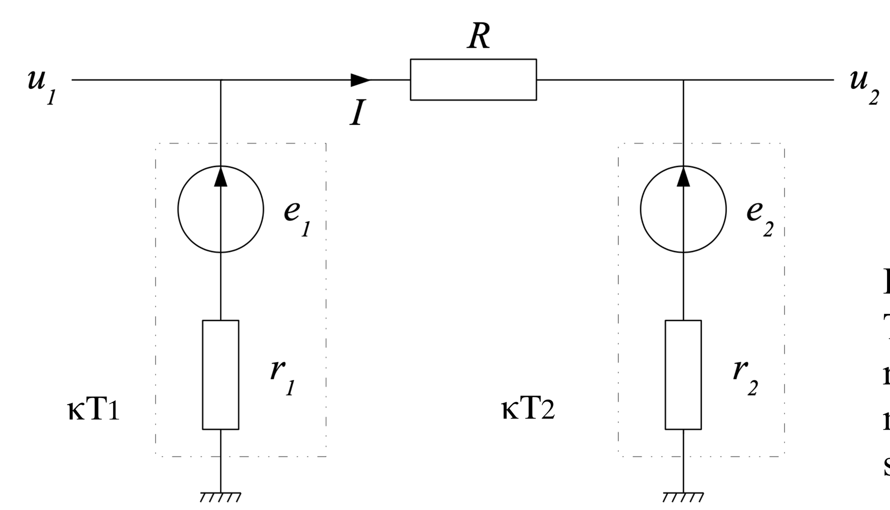


Fig.2: Electrical circuit. The DC-motors are linked by a resistor R. Voltages u_1 and u_2 are measured at the terminals of R, simultaneously.

Current I represents torque, while electromotive force e_1 and e_2 represent velocity of the rotors.
All is measured from voltages $u_1(t)$, $u_2(t)$:

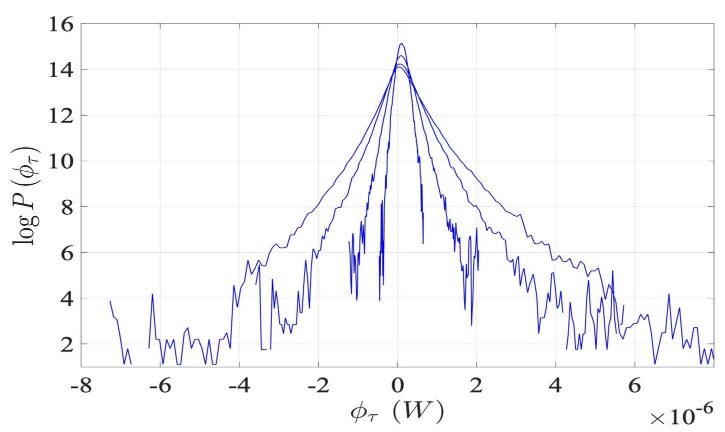
$$I(t) = \frac{u_1(t) - u_2(t)}{R},$$
$$e1(t) = u_1(t)\left(1 + \frac{r_1}{R}\right) - u_2(t)\frac{r_1}{R}$$
$$e2(t) = u_2(t)\left(1 + \frac{r_2}{R}\right) - u_1(t)\frac{r_2}{R}$$

The energy flux can be expressed simply as:

$$\phi(t) = \frac{1}{R + r_1 + r_2}\left(e_1^2(t) - e_2^2(t)\right)$$

On the average, the Fourier law for heat conduction:
$\implies \quad \bar{\phi} \;\propto\; \overline{e_1^2} - \overline{e_2^2} \;\propto\; kT_1 - kT_2$
But, what can be said about the fluctuations?
$\implies \quad \phi(t) \;\; ?$

3. Heat flux from hot to cold... and backward?

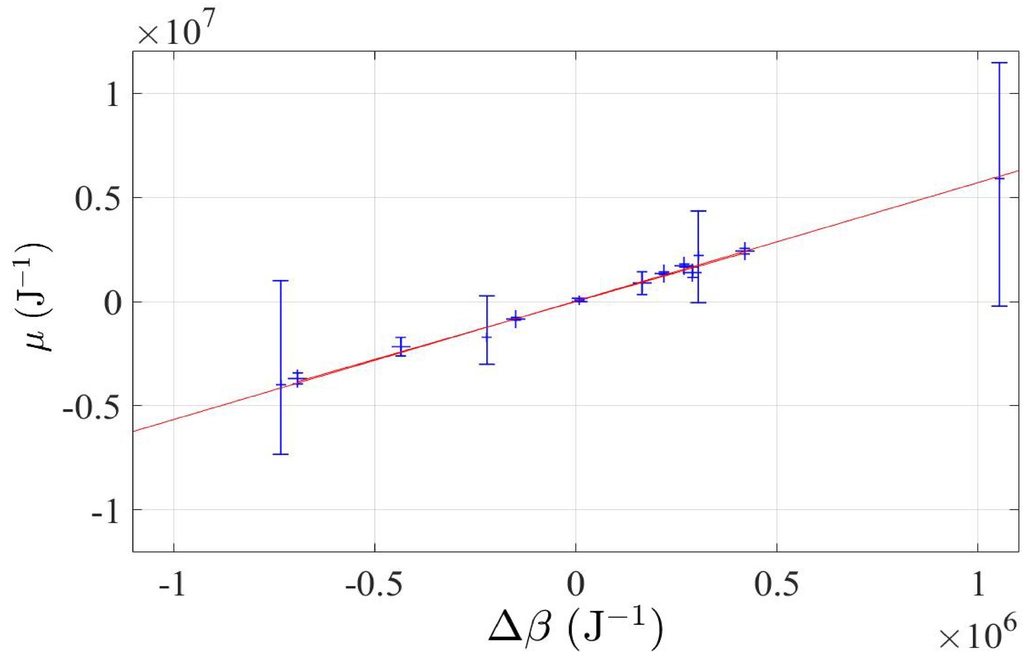


Histograms of the time coarse grained energy flux: $\phi_\tau(t) = \frac{1}{\tau}\int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} \phi(t-t')dt'$

The mean flux is non-zero, but fluctuations are large and asymmetric. Relation between positive and negative fluctuations for large τ we observe that: $\frac{P(\phi_\tau)}{P(-\phi_\tau)} = exp\left(\mu\tau\phi_\tau\right).$

The only free parameter is the exponent : $\mu \neq \Delta\beta$
With:

$$\Delta\beta = \frac{1}{kT_1} - \frac{1}{kT_2}$$



This figure is taken from [1]: the relation between μ and $\Delta\beta$ is linear, for $R = 22\,\Omega$, and the slope is $\frac{\mu}{\Delta\beta} \simeq 5.69$.

4. eXchange Fluctuation Theorem

The eXchange Fluctuation Theorem (XFT) states that the probability for energy ϕ_τ to flow from *hot* to *cold* is exponentially larger than that of the backward flux, during a time $\tau \to \infty$:

$$\frac{P(\phi_\tau)}{P(-\phi_\tau)} = exp\left(\Delta\beta\tau\phi_\tau\right).$$

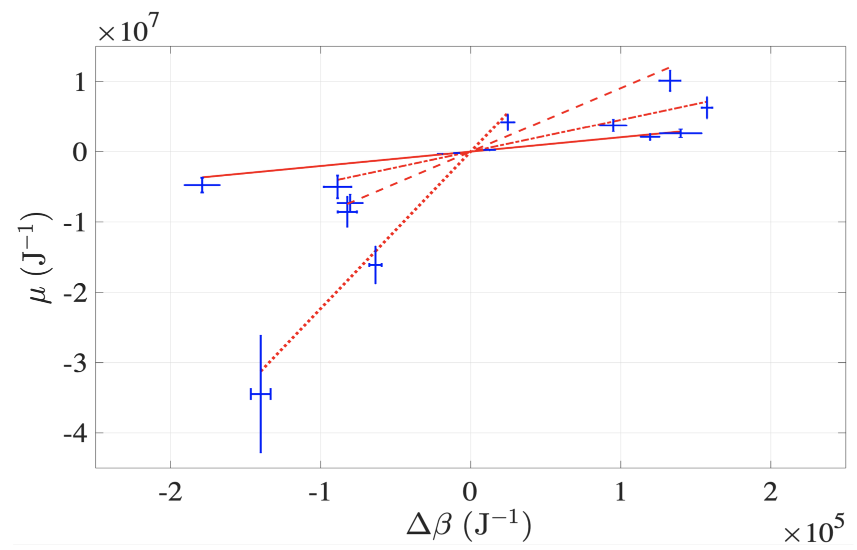
The hot and cold (classical or quantum) heat baths are supposed at equilibrium [2].

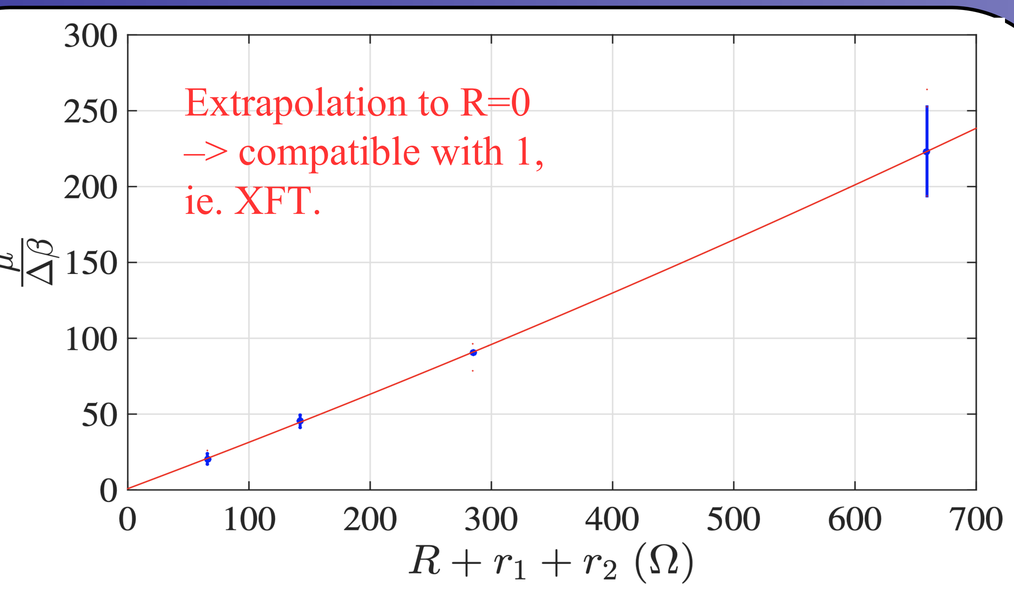
This hypothesis obviously does not hold here!

The observed discrepancy $\mu \neq \Delta\beta$ can be attributed:
– either to the fact that the baths are out-of-equilibrium,
– or to the dissipative nature of the coupling.

5. Vary R

The slope $\frac{\mu}{\Delta\beta}$ depends on R:
A 2$^{\text{nd}}$ order polynomial extrapolation of $\frac{\mu}{\Delta\beta}$ vs $R_{\text{tot.}} = R + r_1 + r_2$ gives the zero-dissipation coupling limit $\frac{\mu}{\Delta\beta} \to 0.85 \simeq 1$:





6. Conclusion

– The XFT is recovered in the $R \to 0$ limit. (Zero 'thermal resistance'.)
– Once again, equilibrium or NESS heat baths play the same role.

7. Openings?

– Continue the study of $\phi(t)$ as a stochastic process (K. Allemang.)
– Smoluchowski-Feynmann ratchet in a NESS heat bath (M. Lagoin, C. Crauste),
– Maxwell daemons → 'Information Thermodynamics',
– Etc.

[1] C-E Lecomte and A. Naert,
Experimental study of energy transport between two granular gas thermostats, JSTAT, **11**, P11004, (2014),
[2] C. Jarzynski and D. K. Wojcik,
Classical and quantum Flucttuation Theorems for Heat Exchange, Phys. Rev. Lett. 92, 23, 230602, 4, (2004),
[3] M. Lameche and A. Naert
Statistical properties of the energy flux between two non-equilibrium steady states thermostats. http://arxiv.org/abs/2101.08632
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