Transient growth of perturbation energy in conservative systems

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When the classical problem of the rotation of a rigid body is considered, it is well-known that a rotation around the axis M_2 corresponding to the intermediate moment of inertia J_2 is unstable. We demonstrate in the present work that a rotation around the two other axis M_1, M_3, which are known to be stable, exhibits transient growth of perturbation energy. This transient growth becomes extremely large if the moment of the stable axis considered (J_1 or J_3) is close to J_2. This initial transient growth of the perturbation leads to an extremely extended nutation of the rotation axis around the considered axis when the initial perturbation is optimized. In the case of a rotation around M_3 with J_3 close to J_2, the leading initital perturbation is along the initial axis M_1. When its initial amplitude M_1 reaches a threshold of order $\sqrt{J_3/J_2 - 1}$, the trajectory changes from a nutation around the M_3 axis toward a trajectory where the initial rotation is periodically transferred from the M_3 to the M_2 direction. The same mechanism of large transient growth of perturbation is generic to any conservative systems and in particular to the triadic resonance in wave systems such as inertial waves.

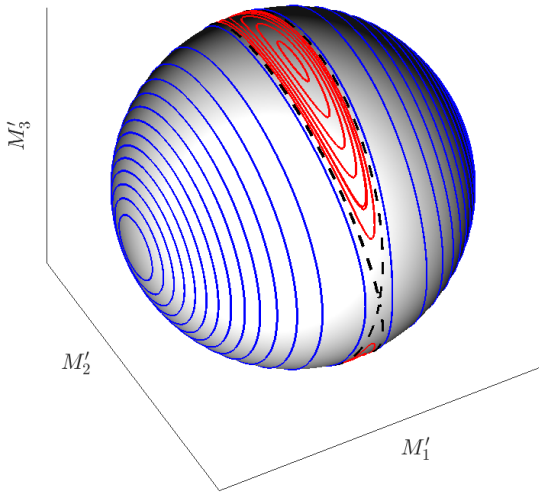

Figure 1. Energy sphere in a (M_1', M_2', M_3')-space with $M_n' = M_n/\sqrt{J_n}$ when $J_3 = J_2(1+\varepsilon)$ with $\varepsilon \ll 1$. The M_1 and M_3 axes are circled by blue and red ellipses which are the line intersections between the energy sphere and the momentum ellipsoid for $\mathcal{M}^2 \in]2J_1\mathcal{E}, 2J_2\mathcal{E}]$ and $\mathcal{M}^2 \in]2J_2\mathcal{E}, 2J_3\mathcal{E}]$ respectively. The lines of equation $\mathcal{M}^2 = 2J_2\mathcal{E}$ represented in dash-dotted dark lines separate the two previous bassins of solutions.

Références

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2. L. Van Damme, P. Mardesic & D. Sugny, The tennis racket effect in a three-dimensional rigid body, *Physica D : Nonlinear Phenomena*, **318**, 17–25 (2017).