Quadrupolar circulation in plane Couette flow

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The transition to turbulence in plane Couette flow is subcritical, characterised by a spatial coexistence of laminar and turbulent domains over a finite range of Reynolds numbers, the transitional flow regime. At the lowest Reynolds number, the coexistence takes the form of localised turbulent spots interspersed amidst otherwise linearly stable laminar base flow. Stimulated by the pioneer direct numerical simulation of Lundbladh and Johansson [1], the modification of plane Couette flow by turbulent spots has been studied both experimentally [2] and numerically [3]. It is found that (i) the wall-normal velocity comprises irregular small-scale fluctuations which decay rapidly away from the spot ; while (ii) large-scale in-plane velocities are slowly varying and direct inward along the streamwise direction whereas outward along the spanwise direction, giving a quadrupolar shape. Moreover, quadrupolar flows have been observed in direct numerical simulations for a model of plane Couette flow with no-slip boundary conditions [4] and for parallel flows with sinusoidal laminar profile and free-slip boundary conditions [5,6], as well as in experiments for plane Poiseuille flow driven by pressure gradient with no-slip boundary conditions [7], indicative that their emergence is generic.

Despite accumulating experimental and numerical observations, the origin of quadrupolar flows is poorly understood and a link between their presence and the Navier-Stokes equation is missing. Using scale analysis, a set of reduced Navier-Stokes equations characterising the spatial scaling of large-scale flows is derived and solved analytically. It is found that the wall-normal velocity is exponentially localised ; while the wall-normal averaged in-plane velocities take the form of a quadruple and show an exponentially truncated algebraic decay. It is noteworthy that the algebraically decaying velocity component is harmonic, i.e. curl-free and divergence-free, and according to the Hodge decomposition theorem, it is unique per Poincaré index of the localised turbulent spot. If there is only one turbulent spot in the flow system, the index of the spot is also unique and solely determined by the Euler characteristic of the flow system, depicting a topological origin of the quadrupolar flow. Consequently, for quasi-two-dimensional parallel flows, i.e. with two spatially extended directions and one highly confined by the walls, we expect that the large-scale flow surrounding a localised turbulent spot is always quadrupolar. Our solution is in good agreement with previous experimental and numerical results.

Références

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