Polynomial expansion of compressible modes in rotating rigid ellipsoids

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Fluid eigenmodes are often used to passively infer physical properties of the interiors of rotating planets and stars [1]. A similar method has been also introduced in fluid dynamics [2]. The experimental technique consists in observing and analysing the splitting in frequency of the acoustic modes, by solving an inverse problem. For instance, the presence of an (unknown) rotational profile disturbs the acoustic spectrum. Moreover, a topographic splitting is often superimposed on the rotational splitting. Indeed, rotating celestial objects are rather ellipsoidal than spherical at the leading order [3], as well as laboratory experiments which are weakly non-spherical (due to mechanical deformations). Rotation and topography should be taken into account simultaneously, but the acoustic problem does not admit exact solutions in the presence of rotation. Fully numerical solutions are often computed [8,9]. However, they cannot be easily combined with inversion schemes to yield robust results. Hence, the usual approach is to consider small perturbations [4] to non-rotating solutions [5,6,7]. As an alternative, we present a new description of the compressible modes, relying on the method of weighted residuals [10]. We introduce an exact spectral Galerkin decomposition of compressible flows, satisfying the vanishing Neumann condition (non-penetration) in triaxial ellipsoids. This decomposition relies on an explicit and global polynomial decomposition in Cartesian coordinates. Within this new framework, we can consistently take into account the global rotation (e.g. the Coriolis force), the compressibility and the ellipsoidal topography. We validate our results against fully numerical simulations performed with the commercial software COMSOL, showing an excellent agreement. Finally, we investigate how the full spectrum of the modes is affected by background density profiles, to consider more realistic models of planetary and stellar interiors.

Références

1. T. L. Duvall Jr, W. A. Dziembowski, P. R. Goode, D. O. Gough, J. W. Harvey, and J. W. Leibacher, *Nature*, **310**(5972) :22, 1984.
2. S. A. Triana, D. S. Zimmerman, H.-C. Nataf, A. Thorette, V. Lekic, and D. P. Lathrop, *New Journal of Physics*, **16**(11) :113005, 2014.
3. S. Chandrasekhar, *Ellipsoidal figures of equilibrium*, Dover Publications, 1969.
4. M. R. Moldover, J. B. Mehl, and M. Greenspan, *The Journal of the Acoustical Society of America*, **79**(2) :253–272, 1986.
5. C. T. M. Chang, *The Journal of the Acoustical Society of America*, **49**(3A) :611–614, 1971.
6. C. T. M. Chang, *The Journal of the Acoustical Society of America*, **51**(1A) :1–5, 1972.
7. M. Willatzen and L. C. Lew Yan Voon, *The Journal of the Acoustical Society of America*, **116**(6) :3279–3283, 2004.
8. F. Lignieres, M. Rieutord, and D. Reese, *Astronomy & Astrophysics*, 455(2) :607–620, 2006.
9. M. Berggren, A. Bernland, and D. Noreland, *Journal of Computational Physics*, **371** :633–650, 2018.
10. B. A. Finlayson, *The method of weighted residuals and variational principles*, SIAM, 2013.