Généralisation de la loi de Darcy dans les milieux poreux pour les fluides à seuils : une approche statistique

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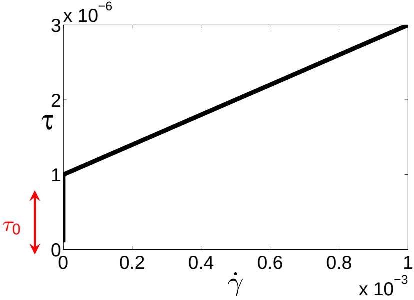
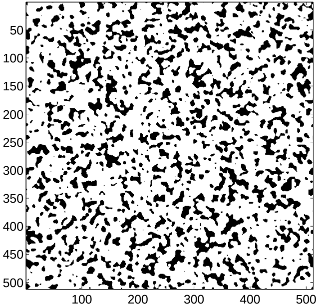
20e RENCONTRE DU NON LINÉAIRE (21-23 mars 2017)

Food, cosmetic, cements, mud, heavy oil...etc

$\dot{\gamma}$: shear rate
τ : shear stress
τ_0: Yield stress
Bingham Fluid:

$$\begin{aligned} \dot{\gamma} &= 0 & \text{si } \tau < \tau_0 \\ \rho\nu_0\dot{\gamma} &= (\tau - \tau_0) & \text{si } \tau > \tau_0 \end{aligned}$$



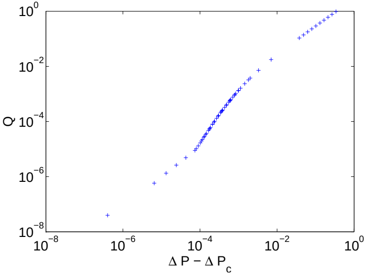


Darcy's Law: $Q = f(\Delta P)$

Litterature

$$\rho \nu_0 \dot{\gamma} = (\tau - \tau_0) \text{ si } \tau > \tau_0$$

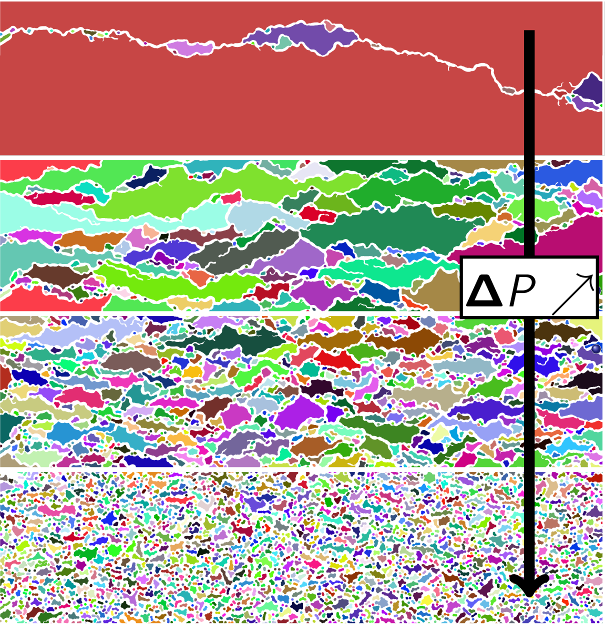
$$Q = K(\Delta P - \Delta P_0) \text{ si } \Delta P > \Delta P_0$$



...But Simulations
(Lattice Boltzmann)

3 scaling regimes

- $Q \propto (\Delta P - \Delta P_c)^1$
- $Q \propto (\Delta P - \Delta P_c)^2$
- $Q \propto (\Delta P - \Delta P_c)^1$



Number of flowing paths increases non-linearly with ΔP

Statistical property of the disorder

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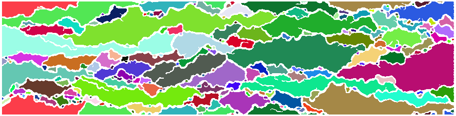
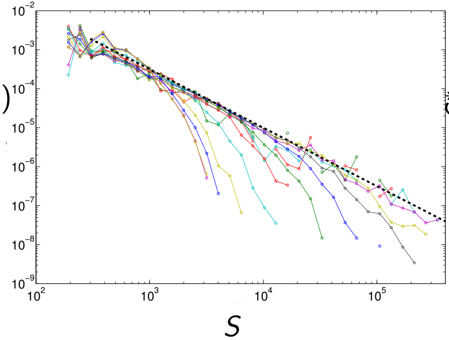
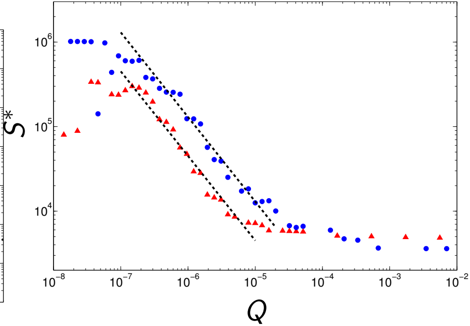


Distribution size exponent τ:



$$p(S) \propto S^{-\tau} \exp -(S/S^*)$$

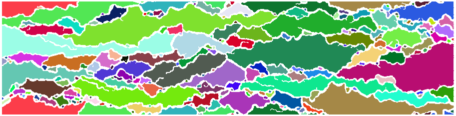
Cut-off exponent γ:



$$S^* \propto (\Delta P - \Delta P_c)^{-\alpha}$$
$$\propto Q^{-\gamma}$$

$$\boxed{p(S) \propto S^{-\tau} \exp -(S/S^*)}$$

$$\boxed{S^* \propto Q^{-\gamma}}$$



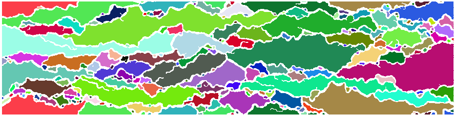
One can predict a scaling law from these exponents:
$Q \propto K\Delta P \propto Q^{\gamma(-\tau+2)}\Delta P$

$$Q^{1-\gamma(-\tau+2)} \propto \Delta P - \Delta P_c, \qquad \tau = 3/2,\ \gamma = 1$$

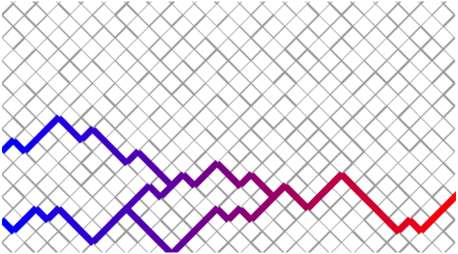
$$\Rightarrow \boxed{Q^{1/2} \propto \Delta P - \Delta P_c}$$

$$p(S) \propto S^{-\tau} \exp -(S/S^*)$$

$$S^* \propto Q^{-\gamma}$$



We now try do understand these scaling laws from simple statistical model: Pore network modeling:



It gives the correct exponent : $\tau = 3/2$, $\gamma = 1$
$\Rightarrow$ Link with statistical phenomena (Avalanches)?
$\Rightarrow$ 3D?
$\Rightarrow$ Other rheology / Disorder ?