Coherent structures generated from random initial conditions in the one-dimensional Nonlinear Schrödinger Equation

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The one-dimensional Nonlinear Schrödinger Equation (1D-NLSE) is well known as a model of wave propagation in nonlinear dispersive media when third-order nonlinearity and second-order dispersion dominate other physical effects. 1D-NLSE could be used to describe waves propagation in plenty of nonlinear systems : nonlinear optical fibers and waveguides, deep water gravitational wave, Bose-Einstein condensate, hot plasma, etc. 1D-NLSE does not yield to general analytical studies and solutions could be found only in some particular cases. In the course of development of this field besides the conventional solitonic solutions, solitons on the finite background (SFB) such as Perigrine soliton [1], Akhmediev breather [2] and Kuznetsov-Ma soliton [3] were found. SFB are the solutions of 1D-NLSE in a focusing case (in the presence of anomalous dispersion). Quite recently such structures were observed in optical and hydrodymical experiments. In these experiments the SFB have been generated by using coherent deterministic initial conditions[4,5,6].

Significant interest to SFB grew up after assumption that rogue wave phenomena observed in the open sea could be described as an interaction of this kind of coherent structures. Rogue waves are waves of extraordinary large height which appears more frequently then predicted by normal low. In nature wind generates water waves through a sophisticated mechanism and the resulting picture is a superposition of random waves. Thereby we underline the importance of stochastic initial conditions in order to use the optical system as laboratory of investigation of hydrodynamical rogue wave phenomena.

Taking into account all aforesaid, in order to investigate a dynamics of stochastic initial conditions it is necessary to use numerical simulations. We applied the split-step Fourier method as the most effective algorithm. In the numerical simulations, we explored a wide range of parameters of stochastic initial conditions. Results of simulations demonstrated occurring of breather like solutions. Detected structures were compared with analytical solutions as well as experimental results.

Références

1. Peregrine D. H., Water waves, nonlinear Schrödinger equations and their solutions, *J. Aust. Math. Soc. Ser.*, **25**, 16–43 (1983).
2. Akhmediev N. & Korneev V. I., Modulation instability and periodic solutions of the nonlinear Schrödinger equation. *Theor. Math. Phys.*, **69**, 1089–1093 (1986).
3. Kuznetsov, E., Solitons in a parametrically unstable plasma. *Sov. Phys. Dokl.*, **22**, 507–508 (1977).
4. Kibler, B., Fatome, J., Finot, C., Millot, G., Dias, F., Genty, G., Akhmediev, N. & Dudley, J. M., The Peregrine soliton in nonlinear fibre optics. *Nature Physics*, **6**, 790–795 (2010).
5. Dudley, J. M., Genty, G., Dias, F., Kibler, B. & Akhmediev, N., Akhmediev Breathers and continuous wave supercontinuum generation. *Opt. Express*, **17**, 21497–21508 (2009).
6. Kibler, B., Fatome, J., Finot, C., Millot, G., Genty, G., Wetzel, B., Akhmediev, N., Dias, F., & Dudley, J. M., Observation of Kuznetsov-Ma soliton dynamics in optical fibre. *Scientific Reports*, **2**, 1–5 (2012).