Critical Transitions in Turbulence : "2D, or not 2D..."

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The properties of two-dimensional (2D) and three-dimensional (3D) hydrodynamical turbulence are known to be quite different. In 3D flows the energy "cascades" to smaller and smaller scales, creating smaller and smaller vortices, eventually dissipating ('forward cascade'). On the other hand, in 2D flows, the energy goes towards larger scales and must be dissipated at these scales ('inverse cascade')[2]. Evidently, these two different properties lead to very different physical situations. A natural question is, then, what happens in the thin-layer limit ? That is, when the flow is very thin so as to approach 2D, but not *exactly* 2D. This situation is very relevant to many physical situations such as atmospheric flow[4,5], protoplanetary disks, the solar tachocline, and possibly other situations. It has been seen that thin layer flows can show a *bidirectional cascade*, meaning that both inverse and forward cascades are present [1,5]. As one varies the thickness of this layer, also denoted as the height, the rate of each respective cascade varies. We want to study this relationship. Moreover, we are interested in looking to see if there are *critical heights* at which the direct (inverse) cascade is exactly zero for some small (large), but finite, thickness, H_{2D} (H_{3D}), and 2D body-forcing. Celani *et al.* have approached some of these questions numerically with a full 3D Navier Stokes direct numerical simulation. They attempted to demonstrate a critical height, but this was not successful since the transition appeared smooth, and they labeled the situation as unresolved [1]. In hopes of answering these questions, we have run high-resolution numerical simulations of a reduced 3D model, complemented with some analytical work (following a process similar to that done in Gallet and Doering [3]). The reduced 3D model, which is essentially a truncation in the z-dependence and therefore two-dimensional, gave us much faster computation times and allowed us to run the high-resolution simulations to a steady state. Apart from being able to do statistics on the results, there are a lot of useful tools in established turbulence theory and phenomenology for steady state turbulence [2]. *With these simulations we confirmed the existence of both H_{2D} and H_{3D}.* Furthermore, the analytical results gave us a conservative and exact bound on H_{2D} as well as a more restrictive, but tentative, bound based on linear stability. Due to the nature of the different critical points, this analytical technique was not applicable for H_{3D} and a theoretical bound for this point is still unknown.

Références

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