Faraday waves revisited

Jean Rajchenbach and Didier Clamond
Université de Nice-Sophia Antipolis- CNRS UMR 7336

Description up to now (following Benjamin Ursell, Proc.Roy.Soc. A225, 505 (1954)

- dispersion relation: $\omega_0^2 = gk\tanh(kh)$ that of free, unforced gravity waves

  k : eigenmode of the tank

mismatch with experiments.

- threshold : $F_{\downarrow} \equiv 4\sigma/\omega$ (σ = dissipation)

bad agreement exp. / theories

- **bifurcation** predicted always surcritical

experiments: **sometimes surcritical, sometimes subcritical.**

- **relation of dispersion** of free gravity waves: **Incompatible with pattern forming in shallow water**. (≠ experiments) *it must be convex !*

Linear analysis

- Relation of dispersion : Floquet theorem + perturbative expansion

$$\omega_0 / \omega \approx 1 \pm \sqrt{(F/4)^2 - (\sigma/\omega)}$$

The relation of dispersion is substantially modified by the dissipation and forcing.

Nonlinear analysis

- the frequency spectrum of water waves is continuous : analysis ≠ of that suited for the parametric pendulum
- **Amplitude equation:** ***the sign of the nonlinear term depends on the depth !***
- **threshold:** $F_{\uparrow} = 4\sqrt{(1-\omega_0/\omega)^2 + (\sigma/\omega)^2}$
- **short waves : surcritical bifurcation.**

  selected wavenumber $2\,\omega_0 = \omega + \sqrt{\omega^2 - 16\,\sigma^2}$
- **long waves : subcritical bifurcation.**

  selected wavenumber $\omega_0 = \omega - 16\,\nu / d^2$