Cascades and Spectra of a Turbulent Spinodal Decomposition – A Case Study in Elastic Turbulence

P.H. Diamond, Xiang Fan; UCSD

L. Chacon, H. Li; LANL

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The Problem

2D Cahn-Hilliard Navier-Stokes (CHNS).

Similar, not identical 2D MHD

2D Cahn-Hilliard, Navier-Stokes

$$\partial_t \psi + \mathbf{v} \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \mathbf{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$$

$$\mathbf{v} = \hat{\mathbf{z}} \times \nabla \phi, \ \omega = \nabla^2 \phi$$

$$\mathbf{B}_\psi = \hat{\mathbf{z}} \times \nabla \psi, \ j_\psi = \xi^2 \nabla^2 \psi$$

2D MHD

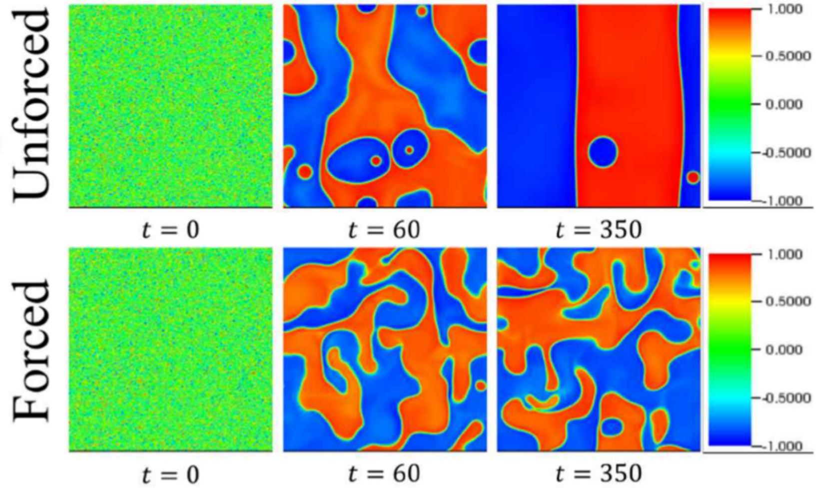
$$\partial_t A + \mathbf{v} \cdot \nabla A = \eta \nabla^2 A$$

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \mathbf{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega$$

$$\mathbf{v} = \hat{\mathbf{z}} \times \nabla \phi, \ \omega = \nabla^2 \phi$$

$$\mathbf{B} = \hat{\mathbf{z}} \times \nabla A, \ j = \frac{1}{\mu_0} \nabla^2 A$$

ψ is density contrast

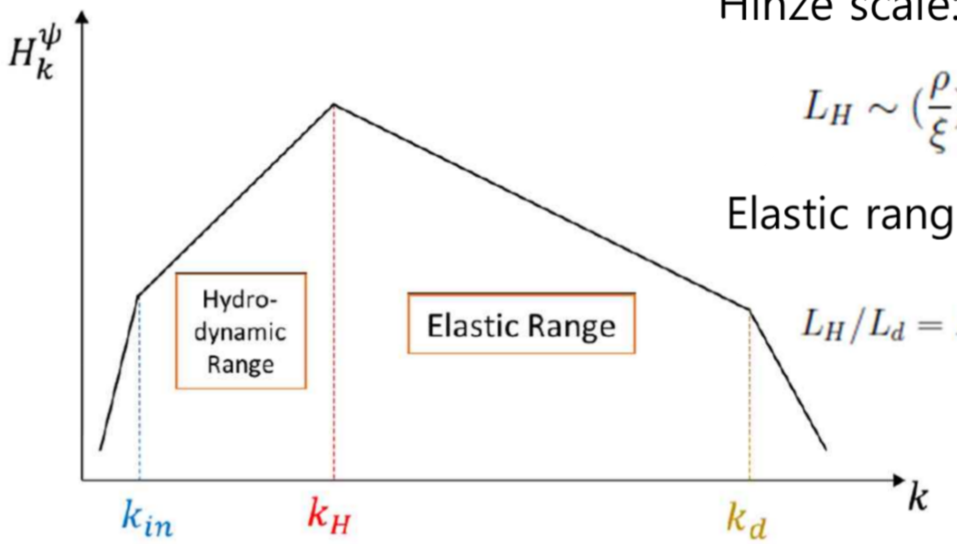


→ Comparison/Contrast

	2D MHD	2D CHNS
Ideal Quadratic Conserved Quantities	Conservation of E, H^A and H^C	Conservation of E, H^ψ and H^C
Role of elastic waves	Alfven wave couples $\mathbf{v}$ with $\mathbf{B}$	CHNS linear elastic wave couples $\mathbf{v}$ with $\mathbf{B}_\psi$
Origin of elasticity	Magnetic field induces elasticity	Surface tension induces elasticity
Origin of the inverse cascades	The coalescence of magnetic flux blobs	The coalescence of blobs of the same species
The inverse cascades	Inverse cascade of H^A	Inverse cascade of H^ψ
Power law of spectra	$H^A_k \sim k^{-7/3}$	$H^\psi_k \sim k^{-7/3}$

Elements of Dynamics

- Elastic range



Hinze scale:

$$L_H \sim (\frac{\rho}{\xi})^{-1/3} \epsilon_\Omega^{-2/9}$$

Elastic range:

$$L_H/L_d = Hd = (\frac{\rho}{\xi})^{-1/3} \nu^{-1/2} \epsilon_\Omega^{-1/}$$

Elastic Waves

$$\omega(\mathbf{k}) = \pm\sqrt{\frac{\xi^2}{\rho}|\nabla\psi_0 \times \mathbf{k}|} - \frac{1}{2}i(CD+\nu)k^2$$

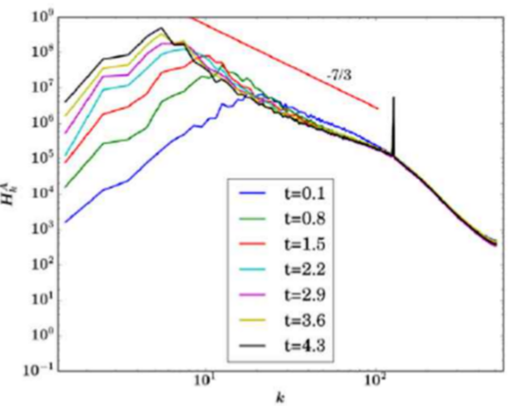
Capillary Wave:

Elastic waves propagate along density interfaces

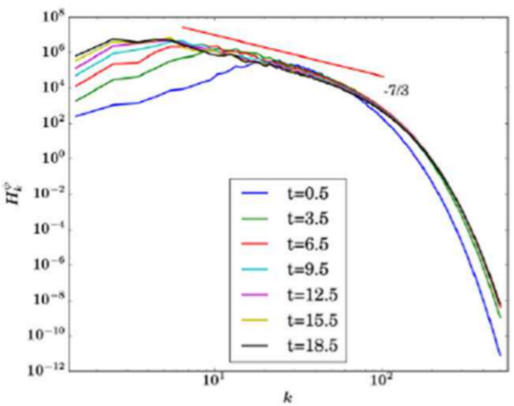
Results

Inverse cascade $\langle\psi^2\rangle$, with $k^{-7/3}$ spectrum

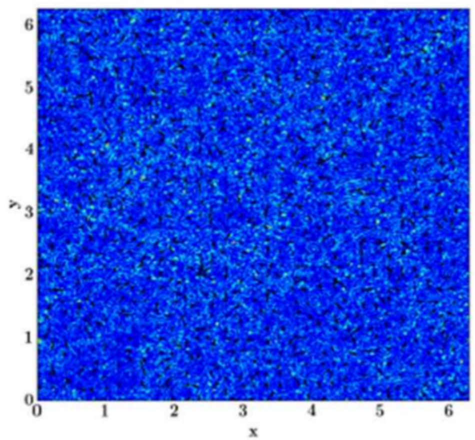
2D MHD



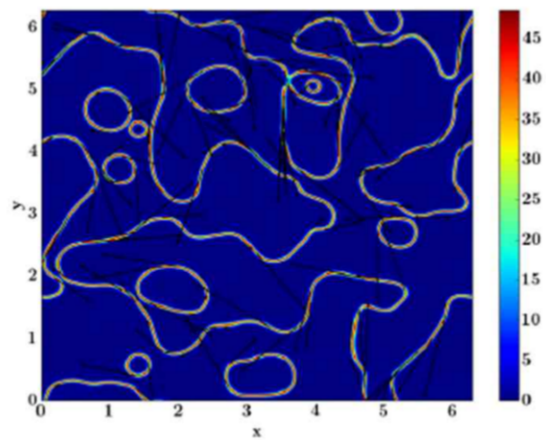
CHNS



$\vec{B}$ field



$\vec{B}_\psi$ field



$E_k \sim k^{-3}$ → more like 2D N-S than 2D MHD??

Packing fraction of B_ψ in CHNS much lower than for B in 2D MHD