Transversal stability of the bouncing ball on a concave surface

J.-Y. Chastaing, G. Pillet, N. Taberlet & J.-C. Géminard

Laboratoire de Physique - ENS de Lyon - CNRS 46, Allée d'Italie, 69007 Lyon - FRANCE
jean-christophe.geminard@ens-lyon.fr

A ball bouncing repeatedly on a vertically-vibrating surface constitutes the famous "bouncing ball" problem, a non-linear system used in the 80's, and still in use nowadays, to illustrate the route to chaos by period doubling [1]. In experiments, in order to avoid the ball escape that would be inevitable with a flat surface, a concave lens is often used to limit the horizontal motion [2]. The expectation is that the slope repels the ball towards the lowest point when it departs from center. We report experimental evidence that the system is, in fact, unstable : the ball experiences a pendular motion which grows with time and then saturates.

We decided to work with the bead locked into the so-called mode 1 : the bead collides with the lens once per period $1/f$. We observed that the bead first moves aside and starts following an almost elliptic trajectory whose main axis slowly rotates. With time, the typical amplitude of the horizontal motion increases in both directions until an almost circular trajectory is reached in the steady state. The typical frequency, f_0, of the pendular motion is observed to be almost constant during the growth of the instability. Moreover, in the whole experimental range, we measured f_0 to be independent of the vibration frequency f and amplitude A. Finally, f_0 depends only on the distance $R - r$ between the center of mass of the bead and the center of curvature of the surface and is very sensible to the physics of the contact.

To characterize the growth of the instability, we measured the distance to the center $D(t)$ as function of time from which we estimated the growth rate σ and steady-state radius of the trajectory D_{max}. We propose theoretical arguments to account for the behaviors of $\sigma(f)$ and of $D_{max}(f)$: the instability grows due to the coupling between the pendular motion and the spin of the bead around an horizontal axis, which is insured by a non-sliding contact at the collision. The growth of the instability is limited by the sliding at the contact during the collision, thus by friction.

Références

1. J. M. Luck and A. Mehta, Bouncing ball with a finite restitution - Chattering, locking and chaos, *Phys. Rev. E* **48**, 3988-3997 (1993).
2. P. Pieránski, Jumping particle model. Period doubling cascade in an experimental system , *J. Physique* **44**, 573-578 (1983).