Exact two-dimensionalization of low-magnetic-Reynolds-number flows subject to a strong magnetic field

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Liquid metal turbulence is encountered in various situations, ranging from metallurgy [1] to the flow in the Earth's outer core [2], including laboratory experiments on magnetohydrodynamic (MHD) turbulence [3,4]. Because their kinematic viscosity is much lower than their magnetic diffusivity, with magnetic Prandtl numbers Pm typically in the range of 10^{-6} to 10^{-5}, flows of liquid metals can be turbulent and still be characterized by a small magnetic Reynolds number Rm : in most laboratory experiments on MHD turbulence, the kinetic Reynolds number is in the range $10^4 - 10^6$, with a magnetic Reynolds number rarely exceeding unity. At small enough scales, turbulence in Earth's outer core may also be considered as a low-Rm flow.

We investigate the behavior of such low-Rm flows, including turbulent flows, driven by a horizontal body-force and subject to a vertical magnetic field, with the following question in mind : for very strong applied magnetic field, is the flow mostly two-dimensional, with remaining weak three-dimensional fluctuations, or does it become *exactly* 2D, with no dependence along the vertical ?

We first focus on the quasi-static approximation, i.e. the asymptotic limit of vanishing magnetic Reynolds number $Rm \to 0$: we prove that the flow becomes exactly 2D asymptotically in time, regardless of the initial condition and provided the interaction parameter N is larger than a threshold value. We call this property *absolute two-dimensionalization* : the attractor of the system is necessarily a (possibly turbulent) 2D flow.

We then consider the full-magnetohydrodynamic equations and we prove that, for low enough Rm and large enough N, the flow becomes exactly two-dimensional in the long-time limit provided the initial vertically-dependent perturbations are infinitesimal. We call this phenomenon *linear two-dimensionalization* : the (possibly turbulent) 2D flow is an attractor of the dynamics, but it is not necessarily the only attractor of the system. Some 3D attractors may also exist and be attained for strong enough initial 3D perturbations.

These results shed some light on the existence of a dissipative anomaly for magnetohydrodynamic flows subject to a strong external magnetic field.

Références

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