Momentum based approximation of incompressible multiphase flows

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Variable density flows and multi-fluid models are important in many applications ranging from magnetohydrodynamics to geophysical flows. We introduce here a new method to approximate the Navier-Stokes equations using the momentum $\mathbf{m} := \rho\mathbf{u}$ with the density, ρ, and the velocity, $\mathbf{u}$, while the fluids interface is tracked via a level set representation. This method has been implemented and tested in the hybrid spectral-finite element code SFEMaNS developed by Guermond and Nore for the past decade [1].

In the frame of variable density problem, the mass matrix associated with the term $\rho\partial_t\mathbf{u}$ need to be recomputed at each time step. The product of ρ and ∂_t can be expensive when using hight-order finite elements, or cannot be made implicit when using spectral method. We propose to use an alternative formulation which involves the momentum, $\mathbf{m} := \rho\mathbf{u}$ as dependant variable for the Navier-Stokes equations. The difficulty now lies in the viscous dissipation that depends on the velocity. To avoid recomputing the mass matrix associated to the dissipation at every time step, this term is treated implicitly. In that order the dissipation operator $-\nabla\cdot(\eta\nabla^s\mathbf{u})$ is rewritten as follows : $-\nabla\cdot(\nu_{\max}\nabla^s\mathbf{m})+\nabla\cdot(\nu_{\max}\nabla^s\mathbf{m}-\eta\nabla^s\mathbf{u})$ where η is the dynamical viscosity. The first term is then made implicit so the correction $\nabla\cdot(\nu_{\max}\nabla^s\mathbf{m}-\eta\nabla^s\mathbf{u})$ can be made explicit. On the other hand, the level set and momentum equations are stabilized with the addition of an artificial viscosity, called viscosity entropy [2]. This viscosity is locally made proportional to the residual of the momentum equation where we define : $\mathrm{Res}_{\mathrm{NS}} = \partial_t\mathbf{m}+\nabla\cdot(\mathbf{m}\otimes\mathbf{u})-\frac{2}{\mathrm{R_e}}\nabla\cdot(\eta\nabla^s\mathbf{u})+\nabla p-\mathbf{f}$. This technique does not perturb the approximation in the regions where the solution is smooth, but it induces diffusion in the regions where the solution experience large gradients.

After implemented this method in SFEMaNS, we start to study a steady stratification of two fluids perturbed with a gravitational wave. The theoretical period of the oscillations of the interface is recovered for various waves perturbations, which confirmed the good behavior of the couple momentum-pressure. The influence of the modeling of the viscous term, with the gradient $\nabla\mathbf{u}$ or with the strain rate tensor $\nabla^s\mathbf{u}$ is studied with a Newton-Bucket set up and a bottom rotating disk experience [3]. Theses tests underline the importance of the strain rate tensor when the dynamical viscosity is variable. To take into account a larger range of problem, a surface tension term is added in SFEMaNS. Its effects are validated with a rising bubble axisymmetric set up [4] for various density and viscosity ratios. A non axisymmetric test-case, which consists of perturbing a bubble interface and computing the oscillation period is also presented. Eventually preliminary results on MHD instabilities involving variable electrical conductivity, such as Tayler (Herreman *et al.* 2015 in review) or metal pad roll instabilities, will be presented.

Références

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