Instanton Trajectories for Random Transitions in Turbulent Flows

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Many turbulent flows undergo sporadic random transitions after long periods of very apparent statistical stationarity. For instance, the Earth's magnetic field reversal or in MHD experiments [1], 2D turbulence experiments [2], atmospheric flows [4] and for paths of ocean currents [5]. This phenomena is far from understood due to the complexity, the large number of degrees of freedom, and the non-equilibrium nature of many of these flows.

A straightforward numerical approach, by direct numerical simulation of the governing equations is just impracticable due the extremely long time between two transitions and to the very high Reynolds number and degrees of freedom number involved. In this talk, we will present an alternative strategy, in which we apply instanton theory.

The transition probability is then represented as a path integral over all possible paths between the two states :

$$P(x,t;x(0)=x_0,x(T)=x_T) \;=\; \int \mathcal{D}[x]\; e^{-\frac{1}{4\alpha}S(x)}. \tag{1}$$

Instanton theory uses the saddle point method in the limit of small noise ($\alpha \to 0$) to show that the most probable transition trajectory (and henceforth the maximizer of the exponential) is the trajectory that minimizes the action $S(x)$ - this trajectory is what is know as the instanton.

In this talk, we present results on applying instanton theory to the 2D Navier-Stokes equations. We show that by minimizing an appropriate action, we can predict the most probable instanton trajectory between two non-equilibrium stationary states and estimate the period of its occurrence. This work is the first application of this promising approach to turbulence problems.

We consider the 2D stochastic Navier-Stokes equations. In a regime of small forces and dissipations, the largest scales of the flow self-organize to produces coherent jets and vortices. Moreover it was recently shown that over long times, random switchings between two non-equilibrium stationary states can occur - more precisely between a parallel and dipole flow [6]. We compute the instanton trajectory and the transition time for observing such a trajectory within the 2D stochastic Navier-Stokes equations and finally, we will discuss the applicability to more complex turbulent flows that show bistability behavior, such as the Kuroshio ocean current [5].

Références

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