A non-linear sigma model to represent two-component Bose-Einstein condensates

Peter Mason[1] & Amandine Aftalion[2]

[1] IJLRDA, UPMC, Paris VI
[2] LMV, Université de Versailles Saint-Quentin en Yvelines
mason@dalembert.upmc.fr

Bose-Einstein condensates possess many interesting and intriguing properties and phenomena. They are new states of matter and represent quantum mechanical effects on a macroscopic scale. In the mean-field regime in which all bosons occupy a single spatial mode, the Bose- Einstein condensate can be described in terms of a macroscopic order parameter. A cubic non-linear Schrödinger equation encapsulates well many aspects of this order parameter, particularly the dynamics and energetic properties. Being inspired by the many experimental works currently produced in the domain, mathematicians and physicists have recently sought to better explain some of the remarkable reported phenomena.

A complete classification of the ground states and topological defects of two-dimensional twocomponent condensates is reported when the orders of the intracomponent coupling strengths, the intercomponent coupling strength and particle numbers are varied and rotation is taken within the physically permissible range. Numerical phase diagrams are produced which show the boundaries between the regions of co-existence, spatial separation and symmetry breaking. Defects such as triangular coreless vortex lattices, square coreless vortex lattices and giant skyrmions are classified. Various aspects of the phase diagrams can be explained in terms of a non-linear σ-model that represents the condensate in terms of the total density.